Solving Rational Equations

Let's think about how to solve rational equations strategically.

7.1 Notice and Wonder: Thoughtful Multiplication

What do you notice? What do you wonder?

$$\frac{3}{x(x-2)} = \frac{2x+1}{x-2}$$

$$\frac{3}{x(x-2)} \cdot x(x-2) = \frac{2x+1}{x-2} \cdot x(x-2)$$

$$3 = 2x^2 + x$$

$$0 = 2x^2 + x - 3$$

7.2 Rational Solving

Jada is working to find values of x that make this equation true: $\frac{5x+5}{x+1} = \frac{5}{x}$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = \text{-}1$, but when I substitute in $x = \text{-}1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

2. Why does Jada's method produce an x value that does not solve the equation?

Are you ready for more?

1. What are the solutions to $x^2 = 1$?

2. What are the solutions to $\frac{x^2}{x-1} = \frac{1}{x-1}$?

3. How can you solve $\frac{x^2}{x-1} = \frac{1}{x-1}$ by inspection?

4. How does the denominator influence the solution(s) to $\frac{x^2}{x-1} = \frac{1}{x-1}$?

5. Why is it important to consider when the denominator is 0?

7.3 More Rational Solving

1. Here are a lot of equations. For each one, use what you know about division to identify values of x that cannot be solutions to the equation.

   a. $\dfrac{x^2+x-6}{x-2} = 5$

   b. $\dfrac{2x+1}{x} = \dfrac{1}{x-2}$

   c. $\dfrac{10}{x+8} = \dfrac{5}{x-8}$

   d. $\dfrac{x^2+x+1}{13} = \dfrac{2}{x-1}$

e. $\dfrac{x+1}{4x} = \dfrac{x-1}{3x}$

f. $\dfrac{1}{x} = \dfrac{1}{x(x+1)}$

g. $\dfrac{x+2}{x} = \dfrac{3}{x-2}$

h. $\dfrac{1}{x-3} = \dfrac{1}{x(x-3)}$

i. $\dfrac{(x+1)(x+2)}{x+1} = \dfrac{x+2}{x+1}$

2. Without solving, identify three of the equations that you think would be least difficult to solve and three that you think would be most difficult to solve. Be prepared to explain your reasoning.

3. Choose three equations to solve. At least one should be from your “least difficult” list and one should be from your “most difficult” list.

Lesson 7 Summary

Consider the equation $\frac{x+2}{x(x+1)} = \frac{2}{(x+1)(x-1)}$. We could solve this equation for x by multiplying each expression by $x(x+1)(x-1)$ to get an equation with no variables in denominators, and then rearranging it into an expression that equals 0. Here is what that looks like:

$$\frac{x+2}{x(x+1)} \cdot x(x+1)(x-1) = \frac{2}{(x+1)(x-1)} \cdot x(x+1)(x-1)$$

$$(x+2)(x-1) = 2x$$

$$x^2 + x - 2 = 2x$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

The last equation, $(x-2)(x+1) = 0$, leads us to believe that the original equation has two solutions: $x = 2$ and $x = \text{-}1$. Substituting $x = 2$ into the original equation, we get $\frac{2+2}{2(2+1)} = \frac{2}{(2+1)(2-1)}$, which is true since each side is equal to $\frac{2}{3}$. But, substituting $x = \text{-}1$ into the original equation, we get $\frac{\text{-}1+2}{\text{-}1(\text{-}1+1)} = \frac{2}{(\text{-}1+1)(\text{-}1-1)}$, which isn't a valid equation since division by 0 is not allowed. This means $x = \text{-}1$ isn't a solution, so what happened to make us think that it was?

Let's consider the simpler equation $x - 5 = 0$. This equation has one solution, $x = 5$. But if we multiply each side by $(x-1)$ the result is a new equation, $(x-1)(x-5) = 0$, which has solutions 5 and 1. The 1 is a solution to the new equation because when $x = 1$, $x - 1 = 0$. But if we substitute 1 for x into the original equation, we get $1 - 5 = \text{-}4 = 0$, which is not a valid equation, so 1 is not a solution to the original equation. Because we multiplied each side of the original equation by an expression that has the value 0 when $x = 1$, the two sides $x - 5$ and 0 that were unequal at that specific x-value are now equal. For this example, $x = 1$ is sometimes called an *extraneous solution.*

In the original example, $x = \text{-}1$ is the extraneous solution. While $x = \text{-}1$ is a solution to the equation we wrote after we multiplied the original equation by $x(x+1)(x-1)$ on each side, it is not a solution to the original equation since they are not equivalent. It should be noted that even though we multiplied by x, $(x+1)$, and $(x-1)$, only one extraneous solution was added. This shows that multiplying by an expression that can equal 0 does not always cause an extraneous solution. So how do we tell if a solution is extraneous or not? We substitute it into the original equation and make sure the result is a valid equation.

- When multiplying each side of an equation by an expression with a variable, we identify what values would make the expression 0 and rule those out as possible solutions.
- Once solutions are found, we substitute them into the original equation and make sure the result is a valid equation.