Unit 8, Lesson 11

What Are Perfect Squares?

Let's see how perfect squares make some equations easier to solve.

11.1 The Thing We Are Squaring

Each of these expressions is a **perfect square**, which means that each can be written as something multiplied by itself.

Rewrite each expression as something multiplied by itself in the form $(_____)^2$. For example, $16x^2$ can be rewritten as $(4x)^2$.

1. 100

2. $9x^2$

3. $\frac{1}{4}x^2$

4. $x \cdot x$

5. $7x \cdot 7x$

6. $(2x - 9)(2x - 9)$

11.2 Perfect Squares in Different Forms

1. Each expression is written as the product of linear factors. Write an equivalent expression in standard form.

   a. $(3x)^2$

   b. $7x \cdot 7x$

   c. $(x+4)(x+4)$

   d. $(x+1)^2$

   e. $(x-7)^2$

   f. $(x+n)^2$

2. Why do you think the following expressions can be described as perfect squares?

$$x^2+6x+9 \qquad x^2-16x+64 \qquad x^2+\tfrac{1}{3}x+\tfrac{1}{36}$$

Are you ready for more?

Expand each expression to have 3 terms, similar to standard form.

1. $(x^2-15)^2$
2. $(\sqrt{x}+7)^2$
3. $(5^x+3)^2$

11.3 Two Methods

Han and Jada solved the same equation with different methods. Here they are:

Han's method:

$$\begin{aligned}(x-6)^2 &= 25\\ (x-6)(x-6) &= 25\\ x^2-12x+36 &= 25\\ x^2-12x+11 &= 0\\ (x-11)(x-1) &= 0\end{aligned}$$

$$x = 11 \quad \text{or} \quad x = 1$$

Jada's method:

$$(x-6)^2 = 25$$

$$x-6 = 5 \quad \text{or} \quad x-6 = -5$$
$$x = 11 \quad \text{or} \quad x = 1$$

Work with a partner to solve these equations. For each equation, one partner solves with Han's method, and the other partner solves with Jada's method. Make sure both partners get the same solutions to the same equation. If not, work together to find your mistakes.

$(y-5)^2 = 49$

$(x+4)^2 = 9$

$(z+\frac{1}{3})^2 = \frac{4}{9}$

$(v-0.1)^2 = 0.36$

Lesson 11 Summary

These are some examples of perfect squares:

- 49, because 49 is $7 \cdot 7$ or 7^2.
- $81a^2$, because it is equivalent to $(9a) \cdot (9a)$ or $(9a)^2$.
- $(x + 5)^2$, because it is equivalent to $(x + 5)(x + 5)$.
- $x^2 - 12x + 36$, because it is equivalent to $(x - 6)^2$ or $(x - 6)(x - 6)$.

A **perfect square** is an expression that is something times itself. Usually we are interested in situations in which the something is a *rational* number or an expression with rational coefficients.

When expressions that are perfect squares are written in factored form and standard form, there is a predictable pattern.

- $(x + 5)(x + 5)$ is equivalent to $x^2 + 10x + 25$.
- $(x - 6)^2$ is equivalent to $x^2 - 12x + 36$.
- $(x - 9)^2$ is equivalent to $x^2 - 18x + 81$.

In general, $(x + n)^2$ is equivalent to $x^2 + (2n)x + n^2$.

Quadratic equations that are in the form $\text{a perfect square} = \text{a perfect square}$ can be solved in a straightforward manner. Here is an example:

$$\begin{aligned} x^2 - 18x + 81 &= 25 \\ (x - 9)(x - 9) &= 25 \\ (x - 9)^2 &= 25 \end{aligned}$$

The equation now expresses that squaring $(x - 9)$ gives 25 as a result. This means $(x - 9)$ must be 5 or -5.

$$\begin{aligned} x - 9 = 5 \quad &\text{or} \quad x - 9 = \text{-}5 \\ x = 14 \quad &\text{or} \quad x = 4 \end{aligned}$$