Unit 8, Lesson 6

# Rewriting Quadratic Expressions in Factored Form (Part 1)

Let's write expressions in factored form.

## 6.1 Puzzles of Rectangles

Here are two puzzles that involve side lengths and areas of rectangles. Can you find the missing area in Figure A and the missing length in Figure B? Be prepared to explain your reasoning.

**Figure A**

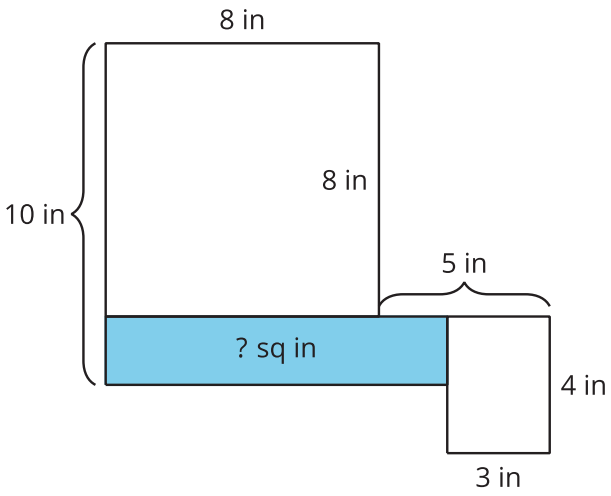
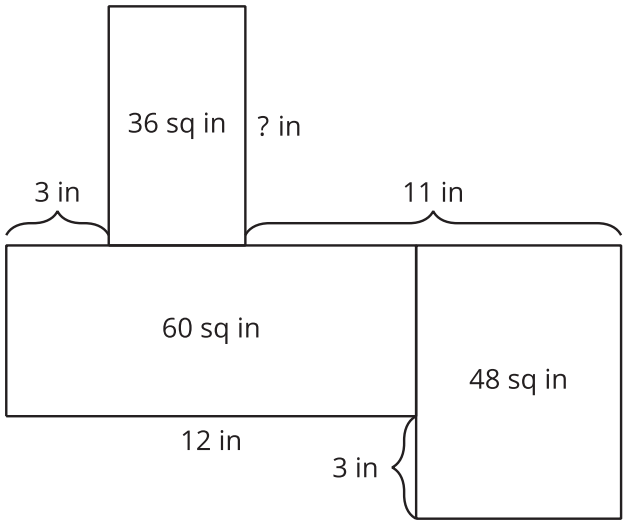



**Figure B**



# 6.2 Using Diagrams to Understand Equivalent Expressions

Compare the diagram that shows $(x + m)(x + n) = x^2 + mx + nx + mn$ and the partially completed diagram for $x^2 + 6x + 8$ to determine the values of $m$ and $n$.

| | $x$ | $n$ |
|---|---|---|
| $x$ | $x^2$ | $nx$ |
| $m$ | $mx$ | $mn$ |

| | $x$ | |
|---|---|---|
| $x$ | $x^2$ | $_____x$ |
| | $_____x$ | 8 |

1. What does the bottom right corner tell you about the connection between the constant term, 8, and the missing parts of the factors, $m$ and $n$?

2. What is the connection between the linear term, $6x$, and the two sections of the diagram with the underlines? How is that connected to $m$ and $n$?

3. Find values for $m$ and $n$, and rewrite $x^2 + 6x + 8$ in factored form.

4. Rewrite these quadratic expressions in factored form. As you work, consider how the values of the constant and linear terms help decide the values of $m$ and $n$.
   a. $x^2 + 3x$
   b. $x^2 - 6x$
   c. $x^2 + 14x + 40$
   d. $x^2 - 6x + 5$
   e. $x^2 - 8x + 12$

# 6.3 Let's Rewrite Some Expressions!

Each row in the table contains a pair of equivalent expressions.

Complete the table with the missing expressions. If you get stuck, consider drawing a diagram.

| factored form | standard form |
|---|---|
| $x(x+7)$ | |
| | $x^2+9x$ |
| | $x^2-8x$ |
| $(x+6)(x+2)$ | |
| | $x^2+13x+12$ |
| $(x-6)(x-2)$ | |
| | $x^2-7x+12$ |
| | $x^2+6x+9$ |
| | $x^2+10x+9$ |
| | $x^2-10x+9$ |
| | $x^2-6x+9$ |
| | $x^2+(m+n)x+mn$ |

## Are you ready for more?

A mathematician threw a party. She told her guests, "I have a riddle for you. I have three daughters. The product of their ages is 72. The sum of their ages is the same as my house number. How old are my daughters?"

The guests went outside to look at the house number. They thought for a few minutes, and then said, "This riddle can't be solved!"

The mathematician said, "Oh yes, I forgot to tell you the last clue. My youngest daughter prefers strawberry ice cream."

With this last clue, the guests could solve the riddle. How old are the mathematician's daughters?

## Lesson 6 Summary

Previously, you learned how to expand a quadratic expression in factored form and write it in standard form by applying the distributive property.

For example, to expand $(x + 4)(x + 5)$, we apply the distributive property to multiply $x$ by $(x + 5)$ and 4 by $(x + 5)$. Then, we apply the property again to multiply $x$ by $x$, $x$ by 5, 4 by $x$, and 4 by 5.

To keep track of all the products, we could make a diagram like this:

| | $x$ | 4 |
|---|---|---|
| $x$ | | |
| 5 | | |

Next, we could write the products of each pair inside the spaces:

| | $x$ | 4 |
|---|---|---|
| $x$ | $x^2$ | $4x$ |
| 5 | $5x$ | $4 \cdot 5$ |

The diagram helps us see that $(x + 4)(x + 5)$ is equivalent to $x^2 + 5x + 4x + 4 \cdot 5$, or in standard form, $x^2 + 9x + 20$.

- The **linear term**, or the term with a single factor of $x$ in the standard form of a quadratic expression, is $9x$ and has a *coefficient* of 9, which is the sum of 5 and 4.
- The *constant term*, 20, is the product of 5 and 4.

We can use these observations to reason in the other direction: starting with an expression in standard form and writing it in factored form.

For example, suppose we wish to write $x^2 - 11x + 24$ in factored form.

Let's start by creating a diagram and writing in the terms $x^2$ and 24.

We need to think of two numbers that multiply to make 24 and add up to -11.

| | $x$ | |
|---|---|---|
| $x$ | $x^2$ | |
| | | 24 |

After some thinking, we see that -8 and -3 meet these conditions. The product of -8 and -3 is 24. The sum of -8 and -3 is -11.

So, $x^2 - 11x + 24$ written in factored form is $(x - 8)(x - 3)$.

| | $x$ | -8 |
|---|---|---|
| $x$ | $x^2$ | $-8x$ |
| -3 | $-3x$ | 24 |