Unit 7, Lesson 20

Beyond Circles

Let's use trigonometric functions to model data.

20.1 Examining Data

Here is some data that we will study in today's lesson.

input	output
1	0.99
2	1.00
3	0.98
4	0.93
5	0.86
6	0.77
7	0.67
8	0.57
9	0.46
10	0.37

input	output
11	0.28
12	0.19
13	0.13
14	0.07
15	0.03
16	0.01
17	0.00
18	0.01
19	0.04
20	0.09

input	output
21	0.16
22	0.24
23	0.33
24	0.43
25	0.54
26	0.65
27	0.76
28	0.85
29	0.92
30	0.98
31	1.00

20.2 Watching the Evening Sky

The data is the amount of the moon that is visible from a particular location on Earth at midnight for each day in January 2018. A value of 1 represents a full moon in which all of the illuminated portion of the moon's face is visible. A value of 0.25 means that one-fourth of the illuminated portion of the moon's face is visible.

1. What is an appropriate midline for modeling the moon data? What about the amplitude? Explain your reasoning.

2. What is an appropriate period for modeling the moon data? Explain your reasoning.

3. Choose a sine or cosine function to model the data. What is the horizontal translation for your choice of function?

4. Propose a function to model the moon data. Explain the meaning of each parameter in your model, and specify units for the input and output of your function.

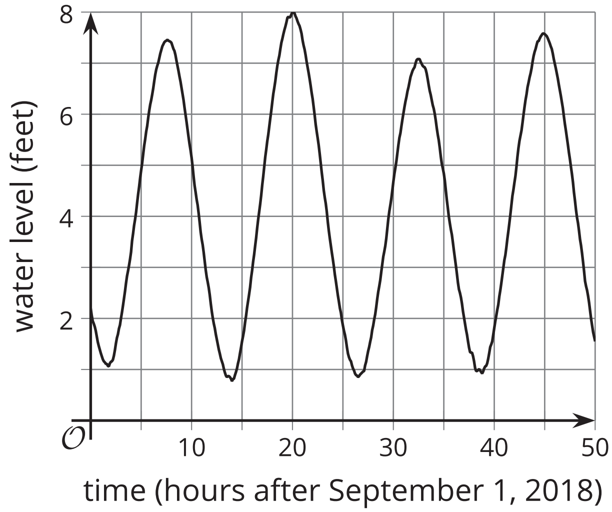
5. Plot the data using graphing technology, and check your choice of parameters (midline, amplitude, period, horizontal translation). What changes did you make to your model?

6. Use your model to predict when the next two full moons will be in 2018. Are your predictions accurate?

7. How much of the moon do you expect to be visible on your birthday? Explain your reasoning.

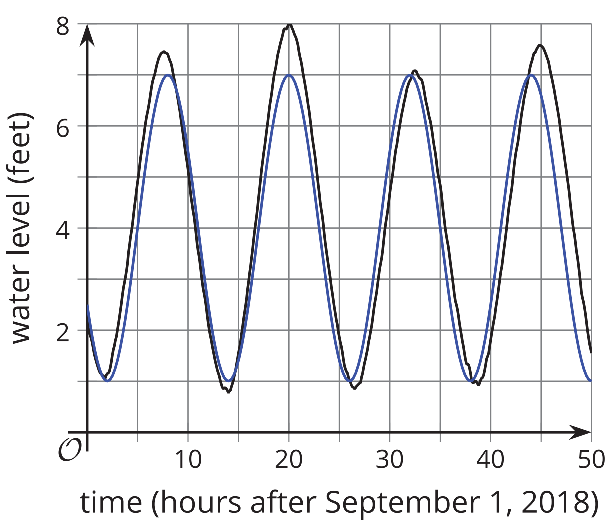
Lesson 20 Summary

Sometimes a phenomenon can be periodic even though it is not connected to motion in a circle. For example, here is a graph of the water level in Bridgeport, Connecticut, over a 50-hour period in 2018.



The midline of the graph appears to be around 4.5 feet. Notice that each day (or each 24-hour period) there are two high tides, a small one where the water goes up by a little less than 3 feet and then a bigger one when the tide goes up by a little more than 3 feet. Since there are two high tides per day, the period for this graph is about 12 hours. The data begins about 1 hour before the tide is at the 4.5 midline value. Since $\sin(0) = 0$, this would make the sine function a good choice for modeling the tide.

Putting together all of our information gives the model $f(h) = 3\sin\left(\frac{2\pi}{12}(h-5)\right) + 4.5$, where h measures hours since midnight on September 1.



Notice that:

- Adding 4.5 gives a midline of $y = 4.5$, which means that the average water height is about 4.5 feet.
- The coefficient of 3 is the amplitude, which averages out the difference in the high tides. In general, the water rises by about 3 feet for each high tide and lowers by about 3 feet for each low tide.
- $\frac{2\pi}{12}$ makes the period 12 hours.
- -5 translates the sine graph to the right by 5 hours, so it has a value of 4.5 at about 5 hours after midnight.