## Lesson 10: Interpreting and Writing Logarithmic Equations

* Let’s look at logarithms with different bases.

### 10.1: Reading Logs

The expression can be read as: “The log, base 10, of 1,000 is 3.”

It can be interpreted as: “The exponent to which we raise a base 10 to get 1,000 is 3.”

Take turns with a partner reading each equation out loud. Then, interpret what they mean.

### 10.2: Base 2 Logarithms

|  |  |
| --- | --- |
| 1 | 0 |
| 2 | 1 |
| 3 | 1.5850 |
| 4 | 2 |
| 5 | 2.3219 |
| 6 | 2.5850 |
| 7 | 2.8074 |
| 8 | 3 |
| 9 | 3.1699 |
| 10 | 3.3219 |

|  |  |
| --- | --- |
| 11 | 3.4594 |
| 12 | 3.5845 |
| 13 | 3.7004 |
| 14 | 3.8074 |
| 15 | 3.9069 |
| 16 | 4 |
| 17 | 4.0875 |
| 18 | 4.1699 |
| 19 | 4.2479 |
| 20 | 4.3219 |

|  |  |
| --- | --- |
| 21 | 4.3923 |
| 22 | 4.4594 |
| 23 | 4.5236 |
| 24 | 4.5850 |
| 25 | 4.6439 |
| 26 | 4.7004 |
| 27 | 4.7549 |
| 28 | 4.8074 |
| 29 | 4.8580 |
| 30 | 4.9069 |

|  |  |
| --- | --- |
| 31 | 4.9542 |
| 32 | 5 |
| 33 | 5.0444 |
| 34 | 5.0875 |
| 35 | 5.1293 |
| 36 | 5.1699 |
| 37 | 5.2095 |
| 38 | 5.2479 |
| 39 | 5.2854 |
| 40 | 5.3219 |

1. Use the table to find the exact or approximate value of each expression. Then, explain to a partner what each expression and its approximated value means.
2. Solve each equation. Write the solution as a logarithmic expression.

### 10.3: Exponential and Logarithmic Forms

These equations express the same relationship between 2, 16, and 4:

1. Each row shows two equations that express the same relationship. Complete the table.

|  | * exponential form | * logarithmic form |
| --- | --- | --- |
| * a. |  |  |
| * b. |  |  |
| * c. |  |  |
| * d. |  |  |
| * e. |  |  |
| * f. |  |  |
| * g. |  |  |
| * h. |  |  |
| * i. |  |  |
| * j. |  |  |

1. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use “?” for the unknown value.
   1. “To what exponent do we raise the number 4 to get 64?”
   2. “What is the log, base 2, of 128?”

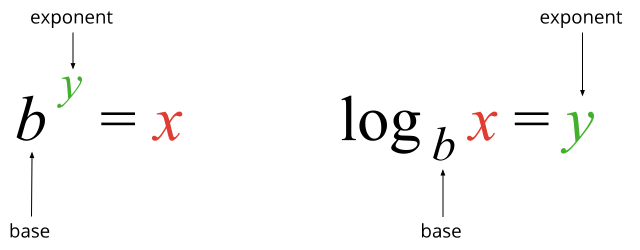
#### Are you ready for more?

1. Is greater than 3 or less than 3? Is greater than or less than 1? Explain your reasoning.
2. How are these two quantities related?

### Lesson 10 Summary

Many relationships that can be expressed with an exponent can also be expressed with a logarithm. Let’s look at this equation: The base is 2 and the exponent is 7, so it can be expressed as a logarithm with base 2:

In general, an exponential equation and a logarithmic equation are related as shown here:



Exponents can be negative, so a logarithm can have negative values. For example , which means that .

An exponential equation cannot always be solved by observation. For example, does not have an obvious solution. The logarithm gives us a way to represent the solution to this equation: . The expression  is approximately 4.25, but is an exact solution.



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