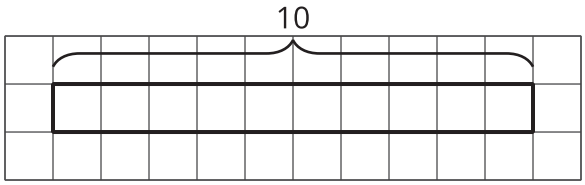
How Many Groups?

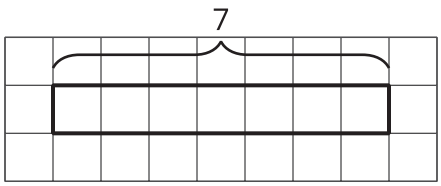
Let's draw tape diagrams to think about division with fractions.

3.1 How Many of These in That?

We can think of the division expression $10 \div 2\frac{1}{2}$ as the question: "How many groups of $2\frac{1}{2}$ are in 10?" Complete the tape diagram to represent this question. Then find the answer.



Complete the tape diagram to represent the question: "How many groups of 2 are in 7?" Then find the answer.

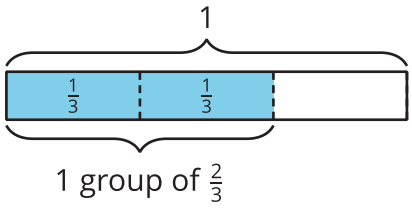


3.2 Representing Groups of Fractions with Tape Diagrams

To make sense of the question "How many $\frac{2}{3}$s are in 1?" Andre wrote equations and drew a tape diagram.

$$? \cdot \frac{2}{3} = 1$$

$$1 \div \frac{2}{3} = ?$$



1. Andre isn't sure how to deal with the remainder.
   - Diego says, "The answer is $1\frac{1}{3}$ because the remainder is $\frac{1}{3}$ of the rectangle."
   - Jada says, "I think it's $1\frac{1}{2}$. Since we want to find out 'how many $\frac{2}{3}$s' there are, we should compare the leftover part to a group of $\frac{2}{3}$. The remainder is $\frac{1}{2}$ of a group."

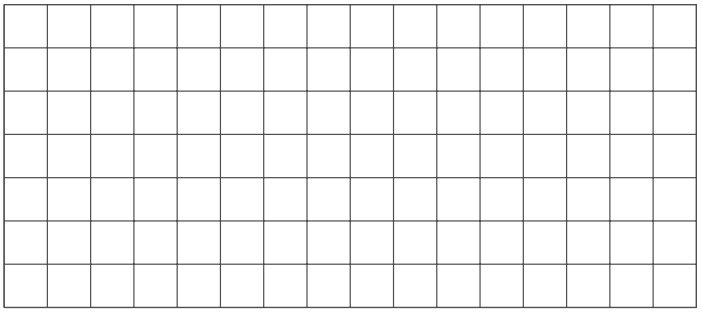
   Do you agree with either of them? Explain or show your reasoning

2. Write a multiplication equation and a division equation for each question. Then draw a tape diagram and find the answer.

   a. How many $\frac{3}{4}$s are in 1?

   b. How many $\frac{2}{3}$s are in 3?

c. How many $\frac{3}{2}$s are in 5?



3.3 Finding the Number of Groups

Write a multiplication equation or a division equation for each question, and then find the answer. Explain or show your reasoning.

1. How many groups of $\frac{1}{2}$ pound are in $2\frac{3}{4}$ pounds?

2. How many $\frac{3}{8}$-inch thick books make a stack that is 6 inches tall?

Are you ready for more?

Write a story with a question that can be represented by the equation $5 \div 1\frac{1}{2} = ?$, and then find the answer. Show your reasoning.

Lesson 3 Summary

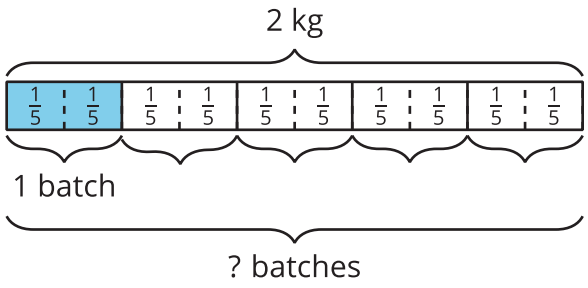
One batch of a recipe calls for $\frac{2}{5}$ kg of flour. If a baker used 2 kg of flour, how many batches did she make?

We can think of the question as "How many groups of $\frac{2}{5}$ kg make 2 kg?" and represent it with the equations:

$$? \cdot \frac{2}{5} = 2$$

$$2 \div \frac{2}{5} = ?$$

To help us make sense of the question, we can draw a tape diagram. This diagram shows 2 whole kilograms, with each kilogram partitioned into fifths.



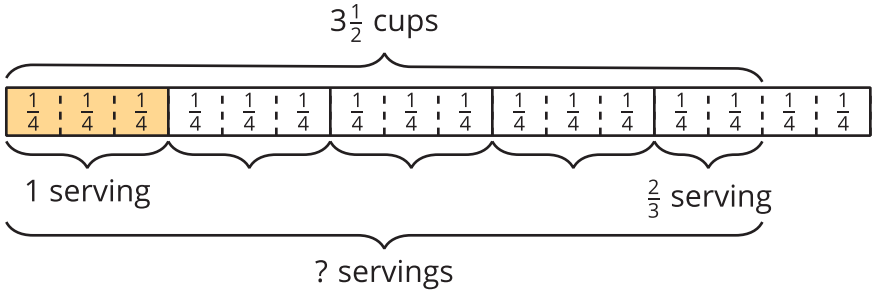
We can see that there are 5 groups of $\frac{2}{5}$ in 2. Multiplying 5 and $\frac{2}{5}$ gives $\frac{10}{5}$ or 2, so $2 \div \frac{2}{5}$ is 5.

Sometimes the number of groups or the result of dividing is not a whole number. Suppose one serving of rice is $\frac{3}{4}$ cup. How many servings are there in $3\frac{1}{2}$ cups?

Here are two equations and a diagram that represent the situation:

$$? \cdot \frac{3}{4} = 3\frac{1}{2}$$

$$3\frac{1}{2} \div \frac{3}{4} = ?$$



The diagram shows 4 full groups of $\frac{3}{4}$, plus 2 extra $\frac{1}{4}$s, which make $\frac{2}{3}$ of a group. So $3\frac{1}{2} \div \frac{3}{4}$ is $4\frac{2}{3}$. We can check this quotient by multiplying $4\frac{2}{3}$ and $\frac{3}{4}$.

$4\frac{2}{3} \cdot \frac{3}{4} = \frac{14}{3} \cdot \frac{3}{4}$, and $\frac{14}{3} \cdot \frac{3}{4} = \frac{14}{4}$, which is $3\frac{2}{4}$ or $3\frac{1}{2}$.