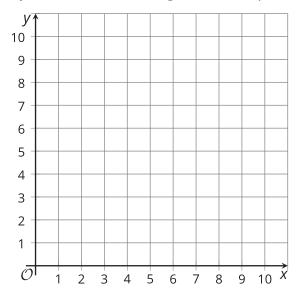
Weighted Averages

Let's split segments using averages and ratios.



Part Way: Points

For the questions in this activity, use the coordinate grid if it is helpful to you.

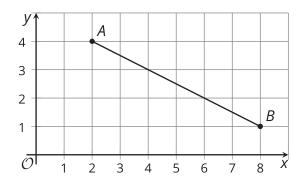


- 1. What is the midpoint of the segment connecting (1,2) and (5,2)?
- 2. What is the midpoint of the segment connecting (5,2) and (5,10)?
- 3. What is the midpoint of the segment connecting (1,2) and (5,10)?



15.2 Part Way: Segment

Point A has coordinates (2,4). Point B has coordinates (8,1).



- 1. Find the point that partitions segment AB in a 2:1 ratio.
- 2. Calculate $C = \frac{1}{3}A + \frac{2}{3}B$.

- 3. What do you notice about your answers to the first 2 questions?
- 4. For 2 new points K and L, write an expression for the point that partitions segment KL in a 3:1 ratio.

Are you ready for more?

Consider the general quadrilateral QRST with Q(0,0), R(a,b), S(c,d), and T(e,f).

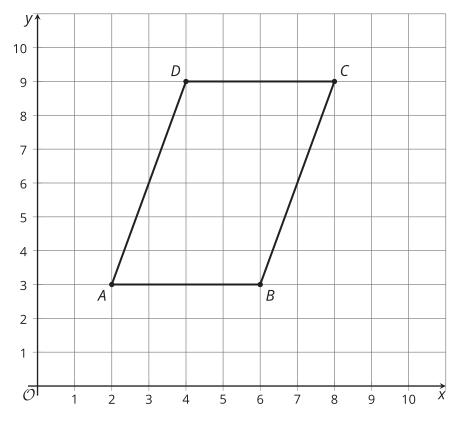
1. Find the midpoints of each side of this quadrilateral.

2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.



15.3 Part Way: Quadrilateral

Here is quadrilateral ABCD.



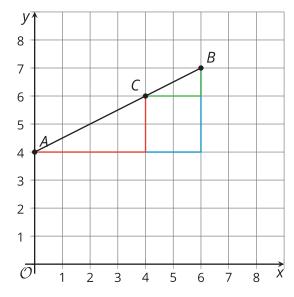
- 1. Find the point that partitions segment AB in a 1:4 ratio. Label it B'.
- 2. Find the point that partitions segment AD in a 1:4 ratio. Label it D'.
- 3. Find the point that partitions segment AC in a 1:4 ratio. Label it C'.
- 4. Is AB'C'D' a dilation of ABCD? Justify your answer.

Lesson 15 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from A(0,4) to B(6,7), average the coordinates of A and B: $\left(\frac{0+6}{2},\frac{4+7}{2}\right)=(3,5.5)$. Another way to write what we just did is $\frac{1}{2}(A+B)$ or $\frac{1}{2}A+\frac{1}{2}B$.

Now, let's find the point that is $\frac{2}{3}$ of the way from A to B. In other words, we'll find point C so that segments AC and CB are in a 2:1 ratio.

In the horizontal direction, segment AB stretches from x=0 to x=6. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point C will be 4 horizontal units away from A, which means an x-coordinate of 4.



In the vertical direction, segment AB stretches from y=4 to y=7. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point C must be 2 vertical units away from A, which means a y-coordinate of 6.

It is possible to do this all at once by saying $C=\frac{1}{3}A+\frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to B than to A. So we give point B more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C=\frac{1}{3}A+\frac{2}{3}B$, substitute and evaluate.

$$\frac{1}{3}A + \frac{2}{3}B$$

$$\frac{1}{3}(0,4) + \frac{2}{3}(6,7)$$

$$\left(0, \frac{4}{3}\right) + \left(4, \frac{14}{3}\right)$$

$$(4,6)$$

Either way, we found that the coordinates of C are (4,6).

