Unit 6 Family Support Materials

Complex Numbers

In this unit, your student will extend what they know about numbers and exponents. They will use familiar exponent rules to see how to evaluate expressions with exponents that are fractions, like $5^{\frac{2}{3}}$. Fractional exponents are related to roots of numbers.

Square roots (written like $\sqrt{16}$ or $16^{\frac{1}{2}}$) and cube roots (written like $\sqrt[3]{8}$ or $8^{\frac{1}{3}}$) will be studied in depth in this unit. Starting from the geometric meaning of square and cube roots, your student will learn to solve equations with variables inside square and cube roots. In geometry, roots are connected to area and volume. For example, if a square has an area of 16 ft^2, then each of its sides is 4 feet long, because 4 is the square root of 16. If a cube has a volume of 8 in^3, then each of its edges is 2 inches long, because 2 is the cube root of 8.

No real numbers (including whole numbers, fractions, negative numbers, and even values like π) can be squared to get a negative number. This means that $\sqrt{-1}$ is not a real number. A new kind of number can be defined as the imaginary number i representing $\sqrt{-1}$.

Here are some tasks to try with your student:

1.
    a. If a square has sides that are 5 feet long, what is the area of the square?
    b. If another square has an area of 20 ft^2, about how long is each of its sides? Try to find an estimate without using a calculator, then check to see how close your estimate was. What would be a better estimate?
2.
    a. If a cube has edges that are 3 meters long, what is its volume?
    b. If another cube has a volume of 30 m^3, about how long is each of its edges? Estimate without a calculator, then check to see how close your estimate was. What would be a better estimate?
3.
    a. If $m^2 = 4$, what could m be? Explain how you know.
    b. If $k^2 = -4$, what could k be? Explain how you know.

Solution:

1.

a. 25 ft^2.

b. A little less than 5 ft, so maybe 4.8 ft. If I square 4.8, I get 23.04, so 4.8 is too big. The exact value can be written as $\sqrt{20}$ which a calculator shows is about 4.472.

2.

a. 27 m^3.

b. A little larger than 3 m, so maybe 3.25 m. If I cube 3.25, I get about 34.33, so 3.25 is too big. The exact value can be written as $\sqrt[3]{30}$ which a calculator shows is about 3.107

3.

a. m could be 2, because $2 \cdot 2 = 4$. m could also be -2, because $-2 \cdot -2$ is also 4.

b. There are no real numbers that could be k because any real number multiplied by itself cannot be negative. Once your student has learned about imaginary numbers, k could be $2i$ and $-2i$. This is because $i = \sqrt{-1}$, so $i^2 = -1$. That means that $(2i)^2 = 4 \cdot i^2 = 4 \cdot -1 = -4$ and $(-2i)^2 = 4 \cdot i^2 = 4 \cdot -1 = -4$.