Combining Bases

Let's multiply expressions with different bases.

7.1 Same Exponent, Different Base

1. Evaluate $5^3 \cdot 2^3$.

2. Evaluate 10^3.

7.2 Power of Products

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? Explain or show your reasoning.

3. What happens when multiplying bases where neither the exponents nor the bases are the same?

7.3 How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. The goal is to write as many expressions as you can that equal a specific number, using any of the exponent rules that we have learned:

$a^n \cdot a^m = a^{n+m}$ $\quad$ $(a^n)^m = a^{n \cdot m}$ $\quad$ $\frac{a^n}{a^m} = a^{n-m}$

$a^n \cdot b^n = (a \cdot b)^n$ $\quad$ $a^0 = 1$ $\quad$ $a^{-n} = \frac{1}{a^n}$

When the time is up, compare your expressions with another group to see how many points you earned.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rules. If the other group has the same expression, neither group earns any points.
- If your *unique* expression uses negative exponents, your group gets 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the exponent rules.

Are you ready for more?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.

2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.

3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Lesson 7 Summary

In this lesson, we developed a rule for combining expressions with the same exponent but different bases: The factors can be regrouped and raised to the same exponent.

Rule	Example showing how it works
$a^n \cdot b^n = (a \cdot b)^n$	$2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$

three factors that are 2 · three factors that are 5 = regrouped factors = three factors that are ten

To see this, expand $2^3 \cdot 5^3$ into three factors that are 2 and three factors that are 5. Regroup the factors into three groups of $2 \cdot 5$, or three groups of 10.