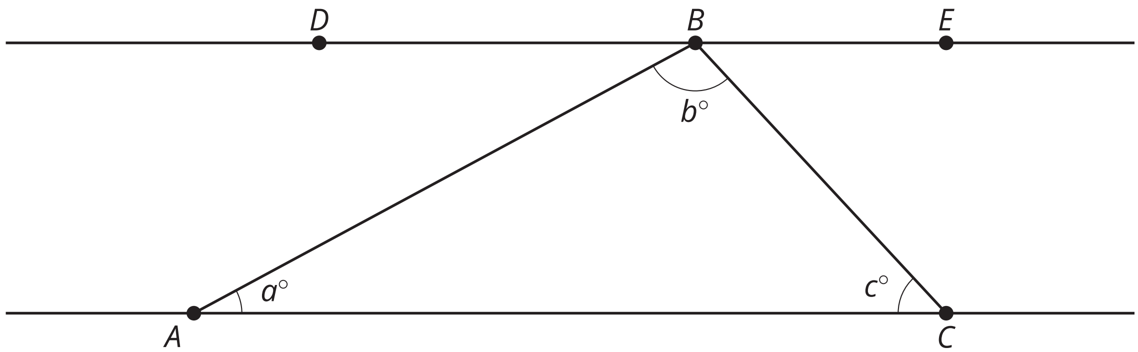
Parallel Lines and the Angles in a Triangle

Let’s see why the angles in a triangle add to 180 degrees.

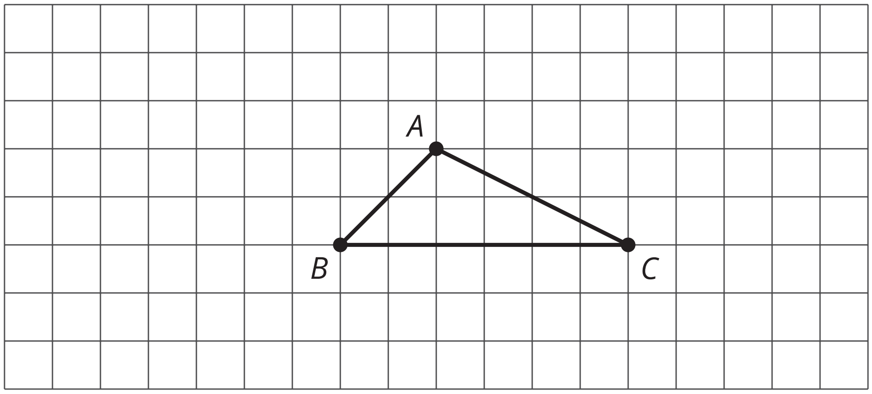
16.1 All the Angles

Here is triangle ABC. Line DE is parallel to line AC.



16.2 Angle Plus Two

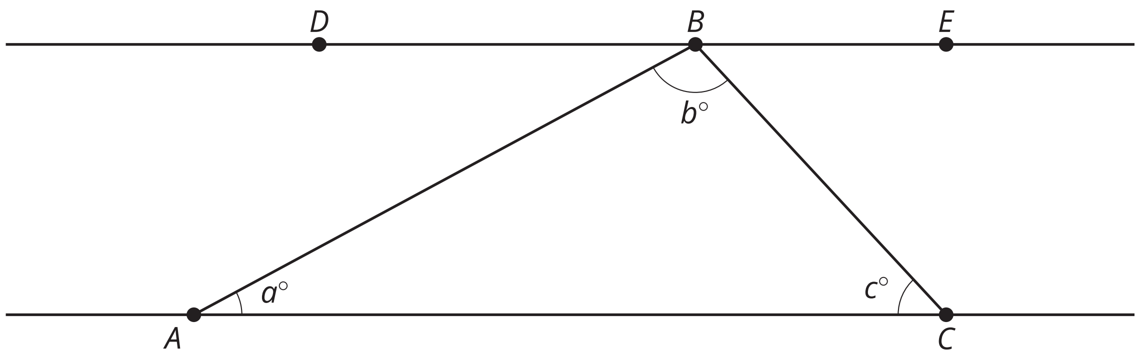
Here is triangle ABC.



1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.
2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.
3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.
4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
6. What is the sum of the measures of angles ABC, BAC, and ACB? Explain your reasoning.

16.3 Every Triangle in the World

Here is triangle ABC. Line DE is parallel to line AC.



1. What is the sum of the measures of angle DBA, angle ABC, and angle CBE?

2. Use your answer to explain why $a + b + c = 180$.

3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is 180°.

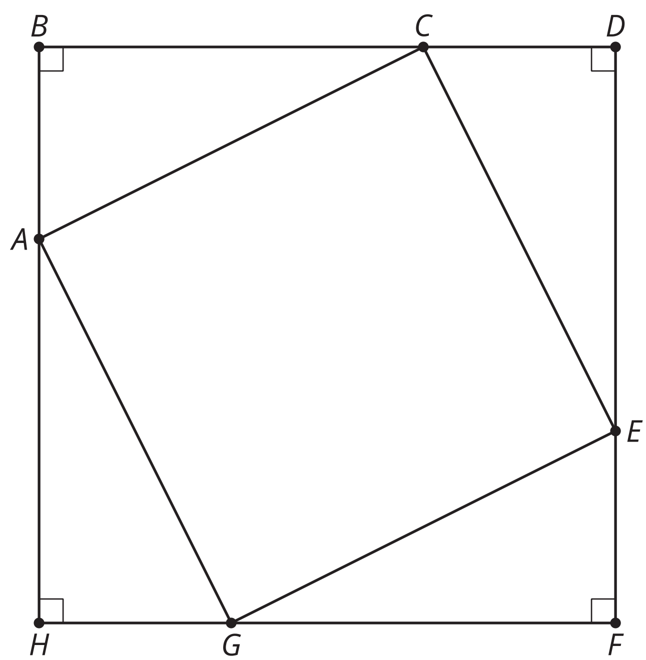
Are you ready for more?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?

2. Come up with an explanation for why anything you notice must be true. (Hint: draw one diagonal in each quadrilateral.)

16.4 Four Triangles Revisited

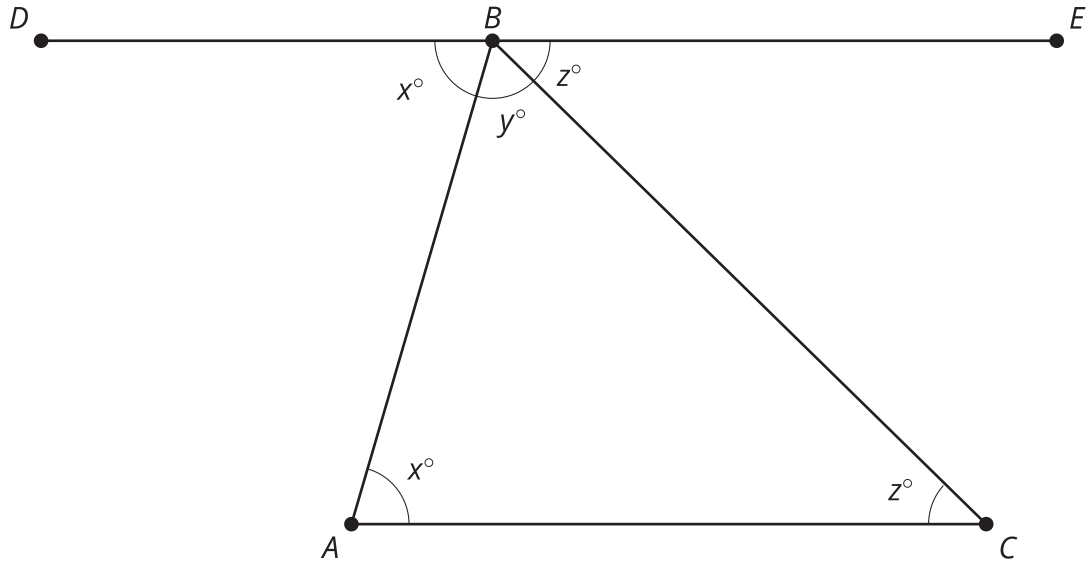
This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.



Given that angle BAC measures 53°, find as many other angle measures as you can.

Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.



A 180° rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure (in the picture these angles are marked as x°).

A 180° rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure (in the picture, these angles are marked as z°).

Also, DBE is a straight line because 180° rotations take lines to parallel lines.

So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in triangle ABC so the sum of the angles in a triangle is always 180°!