Unit 5 Family Support Materials

Proportional Relationships

Section A: Representing Proportional Relationships with Equations

This week your student will learn to write equations that represent proportional relationships. For example, if each square foot of carpet costs \$1.50, then the cost of the carpet is proportional to the number of square feet.

The *constant of proportionality* in this situation is 1.5. We can multiply by the constant of proportionality to find the cost of a specific number of square feet of carpet.

carpet (square feet)	cost (dollars)	
10	15.00	· 1.5
20	30.00	· 1.5
50	75.00	· 1.5

We can represent this relationship with the equation $c = 1.5f$, where f represents the number of square feet and c represents the cost in dollars. Remember that the cost of carpeting is always the number of square feet of carpeting times 1.5 dollars per square foot. This equation is just stating that relationship with symbols.

The equation for any proportional relationship looks like $y = kx$, where x and y represent the related quantities and k is the constant of proportionality. Some other examples are $y = 4x$ and $d = \frac{1}{3}t$. Examples of equations that do not represent proportional relationships are $y = 4 + x$, $A = 6s^2$, and $w = \frac{36}{L}$.

Here is a task to try with your student:

1. Write an equation that represents the relationship between the amounts of grape juice and peach juice in the recipe “for every 5 cups of grape juice, mix in 2 cups of peach juice.”
2. Select **all** the equations that could represent a proportional relationship:
   a. $K = C + 273$
   b. $s = \frac{1}{4}p$

c. $V = s^3$

d. $h = 14 - x$

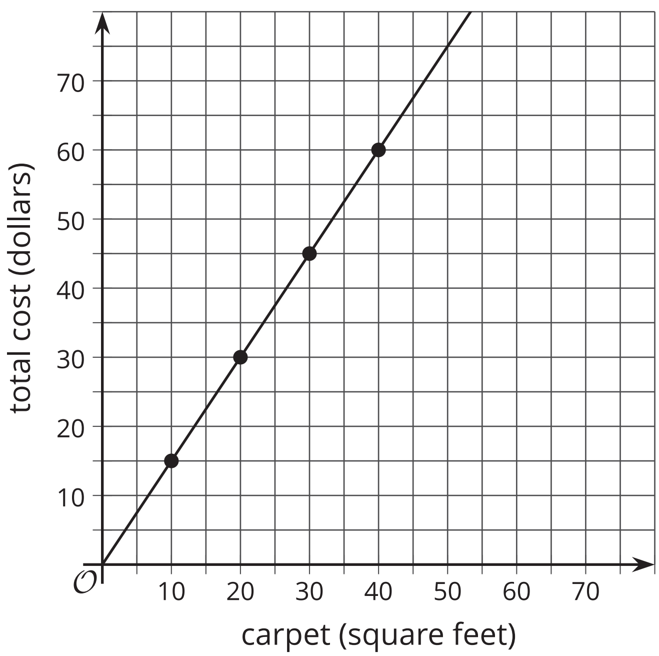
e. $c = 6.28r$

Solution:

1. Answers vary. Sample response: If p represents the number of cups of peach juice and g represents the number of cups of grape juice, the relationship could be written as $p = 0.4g$. Some other equivalent equations are $p = \frac{2}{5}g$, $g = \frac{5}{2}p$, and $g = 2.5p$.
2. B and E. For the equation $s = \frac{1}{4}p$, the constant of proportionality is $\frac{1}{4}$. For the equation $c = 6.28r$, the constant of proportionality is 6.28.

Section C: Representing Proportional Relationships with Graphs

This week your student will work with graphs that represent proportional relationships. For example, here is a graph that represents a relationship between the amount of square feet of carpet purchased and the cost in dollars.

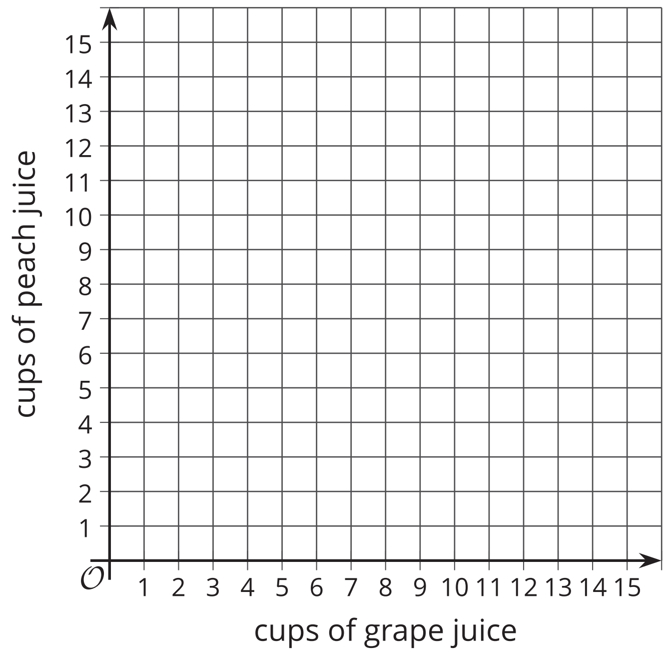


Each square foot of carpet costs $1.50. The point $(10, 15)$ on the graph tells us that 10 square feet of carpet cost $15.

Notice that the points on the graph are arranged in a straight line. If we buy 0 square feet of carpet, it would cost $0. Graphs of proportional relationships are always parts of straight lines including the point $(0, 0)$.

Here is a task to try with your student:

Create a graph that represents the relationship between the amounts of grape juice and peach juice in different-sized batches of fruit juice using the recipe “for every 5 cups of grape juice, mix in 2 cups of peach juice.”


cups of peach juice
cups of grape juice
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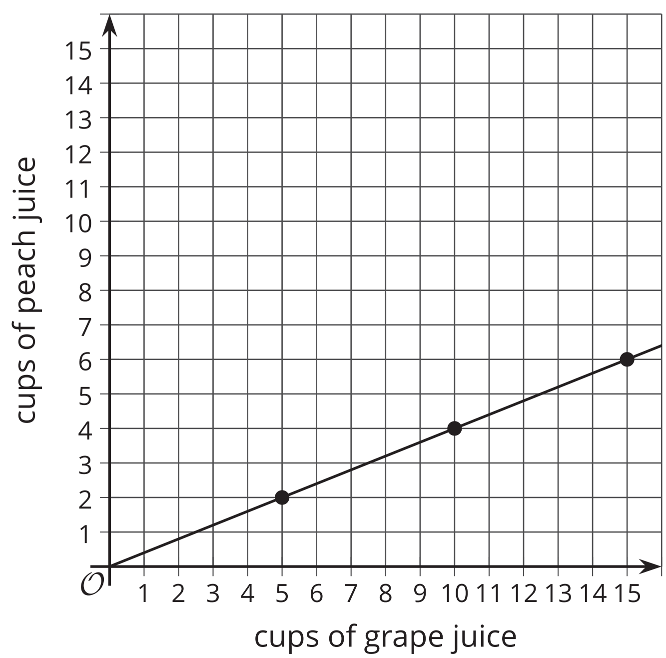
Solution:


cups of peach juice
cups of grape juice
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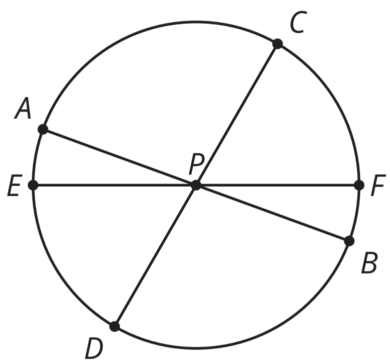
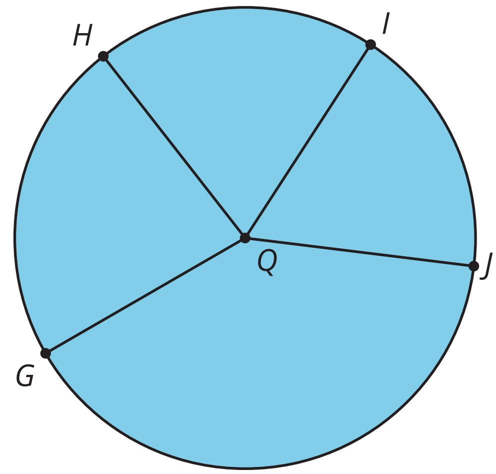
Section D: Circumference of a Circle

This week your student will learn why circles are different from other shapes, such as triangles and squares. Circles are perfectly round because they are made up of all the points that are the same distance away from a center.

Circle 1



Circle 2



- The line segment from the center to a point on the circle is called the **radius**. For example, the segment from P to F is a radius of Circle 1.
- The line segment between two points on the circle and through the center is called the **diameter.** It is twice the length of the radius. For example, the segment from E to F is a diameter of Circle 1. Notice how segment EF is twice as long as segment PF.
- The distance around a circle is called the **circumference**. It is a little more than 3 times the length of the diameter. The exact relationship is $C = \pi d$, where π is a constant with infinitely many digits after the decimal point. One common approximation for π is 3.14.

We can use the proportional relationships between radius, diameter, and circumference to solve problems.

Here is a task to try with your student:

A cereal bowl has a *diameter* of 16 centimeters.

1. What is the *radius* of the cereal bowl?
   - A. 5 centimeters
   - B. 8 centimeters
   - C. 32 centimeters
   - D. 50 centimeters
2. What is the approximate *circumference* of the cereal bowl?

A. 5 centimeters

B. 8 centimeters

C. 32 centimeters

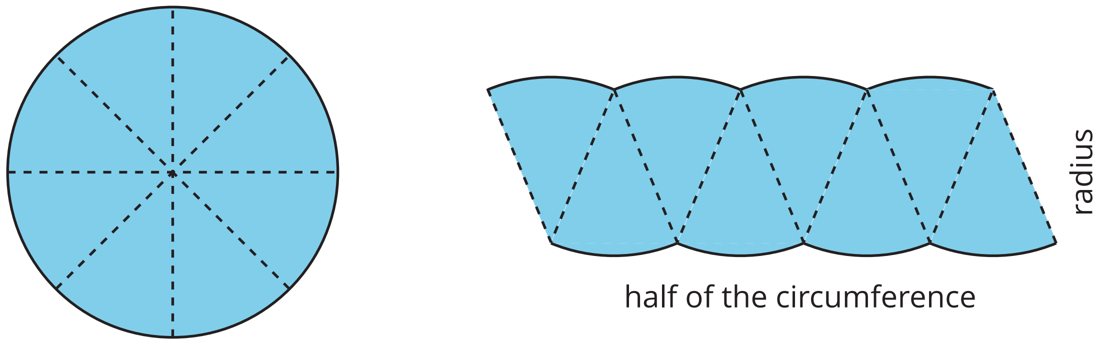
D. 50 centimeters

Solution:

1. B, 8 centimeters. The diameter of a circle is twice the length of the radius, so the radius is half the length of the diameter. We can divide the diameter by 2 to find the radius. $16 \div 2 = 8$.
2. D, 50 centimeters. The circumference of a circle is π times the diameter. $16 \cdot 3.14 \approx 50$.

Section E: Area of a Circle

This week your student will solve problems about the area inside circles. We can cut a circle into wedges and rearrange the pieces without changing the area of the shape. The smaller we cut the wedges, the more the rearranged shape looks like a parallelogram.



The area of a circle can be found by multiplying half of the circumference times the radius. Using $C = 2\pi r$ (which is equivalent to $C = \pi d$ because $2r = d$) we can represent this relationship with the equation:

$$A = \frac{1}{2}(2\pi r) \cdot r$$

Or

$$A = \pi r^2$$

This means that if we know the radius, we can find the area. For example, if a circle has radius 10 cm, then the area is about 314 cm^2, because $3.14 \cdot 10^2 = 314$. We can also say that the area is 100π cm^2.

Here is a task to try with your student:

A rectangular wooden board, 20 inches wide and 40 inches long, has a circular hole cut out of it.

1. The diameter of the circle is 6 inches. What is the area?
2. What is the area of the board after the circle is removed?

Solution:

1. 9π, or about 28.26 in^2. The radius of the hole is half of the diameter, so we can divide to get the radius, $6 \div 2 = 3$. The area of a circle can be calculated as $A = \pi r^2$. For a radius of 3, we get $3^2 = 9$ for r^2. We can write 9π or use 3.14 as an approximation of pi, which gives us $3.14 \cdot 9 = 28.26$.
2. $800 - 9\pi$ or about 771.74 in^2. Before the hole was cut out, the entire board had an area of $20 \cdot 40$, or 800, in^2. We can subtract the area of the missing part to get the area of the remaining board, $800 - 28.26 = 771.74$.