Scope and Sequence for Algebra 2

Students begin the course with a study of sequences, which is also an opportunity to revisit linear and exponential functions. Students represent functions in a variety of ways while addressing some aspects of mathematical modeling. This work leads students to analyze situations that are well modeled by polynomials before pivoting to study the structure of polynomial graphs and equations. Students do arithmetic on polynomials and rational functions and use different forms to identify asymptotes and end behavior. Students also study polynomial identities and use some key identities to establish the formula for the sum of the first n terms of a geometric sequence.

Next, students extend exponent rules to include rational exponents. Students solve equations involving square and cube roots before developing the idea of i, a number whose square is -1. The number i expands the number system to include complex numbers and allows students to solve quadratic equations with non-real solutions.

Building on rational exponents, students return to their study of exponential functions and establish that the property of growth by equal factors over equal intervals holds even when the interval has non-integer length. Students use logarithms to solve for unknown exponents, and are introduced to the number e and its use in modeling continuous growth. Logarithm functions and some situations they model well are also briefly addressed.

Students learn to transform functions graphically and algebraically. In previous courses and units, students adjusted the parameters of particular types of models to fit data. In this course, students consolidate and generalize this understanding. This work is useful in the study of periodic functions that comes next. Students work with the unit circle to make sense of trigonometric functions, and then students use trigonometric functions to model periodic relationships.

The last unit, on statistical inference, focuses on analyzing experimental data modeled by normal distributions. Students learn to use sampling and simulations to account for variability in data and estimate population mean, margin of error, and proportions. Students develop skepticism about news stories that summarize data inappropriately.

Modeling prompts are provided for use throughout the course. While students have opportunities to engage in aspects of mathematical modeling during class activities, modeling prompts allow students to engage in the full modeling cycle. Modeling prompts can be implemented in various ways. Please see the *Mathematics Modeling Prompts* section of this Course Guide for a more detailed explanation.

Unit 1: Sequences and Functions

This unit provides students an opportunity to revisit functions by way of sequences. Through many concrete examples, students will see arithmetic and geometric sequences as linear and exponential functions restricted to a domain that is a subset of the integers. This unit reinforces understanding of functions by using multiple representations (including graphs, tables, and expressions).

Students begin with an invitation to describe sequences with informal language and learn to identify if a sequence is geometric, arithmetic, or neither. Students write out the terms of sequences arising from mathematical situations, in addition to interpreting and creating tables and graphs about the given relationship.

Students learn that sequences are a type of function in which the input variable is the position and the output variable is the term at that position. They learn to interpret sequences and then use function notation to write their own recursive definitions of the sequences. Students make connections between the sequences and different representations of functions.

Students then build on their prior knowledge of linear and exponential functions to develop explicit formulas for arithmetic and geometric sequences and to model several situations represented in different ways.

In the last section of the unit, students use sequences to model several situations represented in different ways. This work is meant to touch on some practices that must be attended to while modeling, such as choosing a good model and identifying an appropriate domain. Students also recognize that a sequence is an appropriate type of function to use as a model for these situations since the domain of each situation is a subset of the integers. Finally, students encounter some situations in which it makes sense to compute the sum of a finite sequence. This thinking connects to later work in which students will determine the formula for the sum of the first n terms of a geometric sequence.

Section A: Sequences

- Lesson 1: A Towering Sequence
- Lesson 2: Introducing Geometric Sequences
- Lesson 3: Different Types of Sequences
- Lesson 4: Using Technology to Work with Sequences

Section B: Representing Sequences

- Lesson 5: Sequences Are Functions
- Lesson 6: Representing Sequences
- Lesson 7: Representing More Sequences

Section C: What’s the Equation?

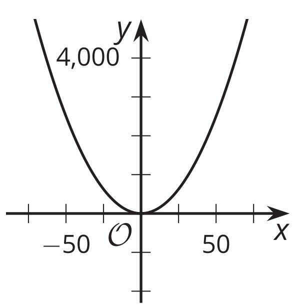
- Lesson 8: The n^{th} Term
- Lesson 9: What’s the Equation?
- Lesson 10: Situations and Sequence Types
- Lesson 11: Adding Up

Unit 2: Polynomial Functions

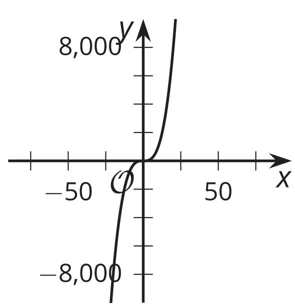
This unit extends students’ previous work with linear and quadratic functions as they investigate polynomials of higher degree. Students rewrite polynomials in different forms, recognizing the benefits of the various forms for their ability to reveal the structure of key features of their graphs.

The unit begins with an introduction to two situations that can be modeled by a polynomial function. Students build their understanding of what polynomials are and what their graphs can look like. Certain aspects, such as end behavior, will be important in a later unit when students explore the end behavior of rational functions.

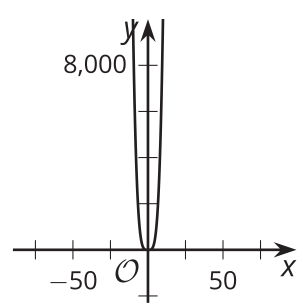
$y = x^2$



$y = x^3$



$y = x^4$



Focusing on functions expressed in factored form and their graphs, students connect that a factor of $(x - a)$ means a is a zero of the function and $(a, 0)$ is a horizontal intercept. The effect of the degree and leading coefficient on end behavior is established along with the effect of multiplicity on the shape of the graph near zeros of the function. Taking in all of these features, students learn to sketch polynomial functions expressed as a product of linear factors.

In a previous course, students used the distributive property to multiply factors, and also factored quadratics. Opportunities to review these skills and apply them to polynomials of higher degree are embedded throughout the unit and in practice problems. This prepares students for the final section where students divide a polynomial written in standard form by a suspected factor. From there, the connection between division and multiplication equations is used to establish the Remainder Theorem. This allows the conclusion that if a polynomial has a zero at $x = a$, then it must also have $(x - a)$ as a factor.

Section A: What Is a Polynomial?

- Lesson 1: Let's Make a Box
- Lesson 2: Funding the Future
- Lesson 3: Introducing Polynomials
- Lesson 4: Combining Polynomials

Section B: Working with Polynomials

- Lesson 5: Connecting Factors and Zeros
- Lesson 6: Different Forms
- Lesson 7: Using Factors and Zeros

Section C: Graphs of Polynomials

- Lesson 8: End Behavior (Part 1)
- Lesson 9: End Behavior (Part 2)
- Lesson 10: Multiplicity
- Lesson 11: Finding Intersections

Section D: Polynomial Division

- Lesson 12: Polynomial Division (Part 1)
- Lesson 13: Polynomial Division (Part 2)
- Lesson 14: What Do You Know about Polynomials?
- Lesson 15: The Remainder Theorem

Unit 3: Rational Functions and Equations

Building on the "Polynomial Functions" unit, in this unit, students transition to working with rational functions, rational equations, and identities.

The unit begins with an introduction to rational functions as students consider situations they can model, such as when minimizing surface area or determining average costs. Students examine the asymptotic behavior of their graphs, which relates to the structure of the equations. Students continue to build on structure as they use polynomial division to rewrite rational expressions for the purpose of identifying the end behavior of the function. Students then focus on solving rational equations and making sense of how some methods can lead to possible solutions that are in fact not solutions (so-called "extraneous solutions").

In the final section, students study identities. Students sharpen their skills manipulating expressions while proving, or disproving, that two expressions are equivalent. The unit concludes with a return to geometric sequences first examined in the previous unit and, using a polynomial identity proved at the start of the section, students derive the formula for the sum of a finite geometric series before using the formula to solve problems.

Section A: Rational Functions

- Lesson 1: Minimizing Surface Area
- Lesson 2: Graphs of Rational Functions (Part 1)
- Lesson 3: Graphs of Rational Functions (Part 2)
- Lesson 4: End Behavior of Rational Functions

Section B: Rational Equations

- Lesson 5: Rational Equations (Part 1)
- Lesson 6: Rational Equations (Part 2)
- Lesson 7: Solving Rational Equations

Section C: Polynomial Identities

- Lesson 8: Polynomial Identities (Part 1)
- Lesson 9: Polynomial Identities (Part 2)
- Lesson 10: Summing Up
- Lesson 11: Using the Sum

Unit 4: Complex Numbers and Rational Exponents

In this unit, students use what they know about exponents and radicals to extend exponent rules to include rational exponents, solve equations involving squares and square roots, develop an understanding of complex numbers, and solve quadratic equations that include complex roots.

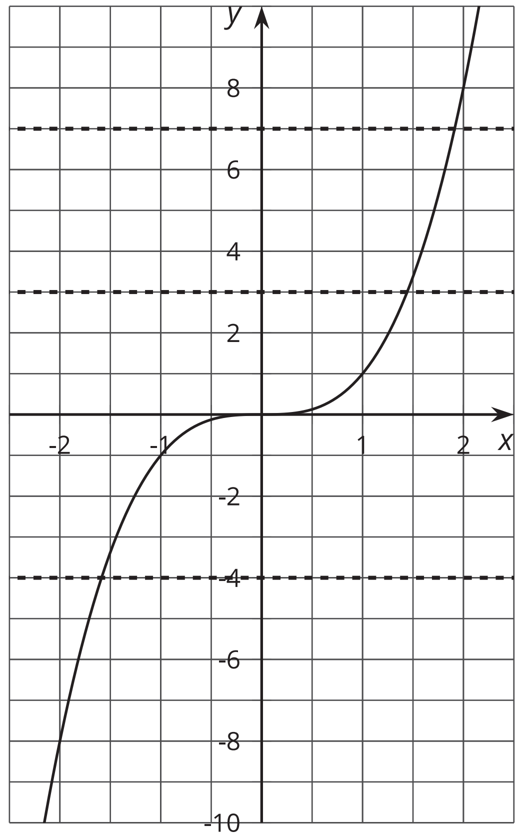
The unit opens with an optional review of exponent rules before using those rules to justify why $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ when a and b are integers and x is positive. The new rule leads to an exploration of square and cube roots as solutions to equations of the form $x^2 = c$ or $x^3 = c$. In particular, students learn that positive numbers have two square roots and that $\sqrt{c}$ represents only the positive root.

Students explore equations involving square and cube roots to recognize that squaring each side of an equation can introduce solutions that are not solutions to the original equation.

Note that there are claims in the unit like, “When a square is equal to a negative number, there are no solutions,” which would be more precisely stated as “When a square is equal to a negative number, there are no real number solutions.” However, until students learn about imaginary numbers in the next section, it doesn't make sense to make this distinction.

The third section introduces imaginary and complex numbers by proposing i as a solution to the equation $x^2 = -1$. Students explore the implications of this new type of number by representing it on an imaginary axis off of the real number line. This exploration includes adding imaginary and real numbers together to get complex numbers, and then adding and multiplying complex numbers.

Next, students use their new understanding of complex numbers to solve more quadratic equations. Finally, there is an optional exploration of data representation on a map using circles with area proportional to the data, in which students must use their understanding of square roots to find the radii of the appropriate circles.



Section A: Exponent Properties

- Lesson 1: Properties of Exponents
- Lesson 2: Square Roots and Cube Roots
- Lesson 3: Exponents That Are Unit Fractions
- Lesson 4: Positive Rational Exponents
- Lesson 5: Negative Rational Exponents

Section B: Solving Equations with Square and Cube Roots

- Lesson 6: Squares and Square Roots
- Lesson 7: Inequivalent Equations?
- Lesson 8: Cubes and Cube Roots
- Lesson 9: Solving Radical Equations

Section C: A New Kind of Number

- Lesson 10: A New Kind of Number
- Lesson 11: Introducing the Number i
- Lesson 12: Arithmetic with Complex Numbers
- Lesson 13: Multiplying Complex Numbers
- Lesson 14: More Arithmetic with Complex Numbers

- Lesson 15: Working Backward

Section D: Solving Quadratics with Complex Numbers

- Lesson 16: Solving Quadratics
- Lesson 17: Completing the Square and Complex Solutions
- Lesson 18: The Quadratic Formula and Complex Solutions
- Lesson 19: Real and Non-Real Solutions

Section E: Let's Put It to Work

- Lesson 20: Drawing Proportional Circles

Unit 5: Exponential Functions and Equations

In this unit, students explore exponential and logarithmic functions. The unit begins with students recalling geometric sequences and drawing a connection to the growth or decay of values by a constant growth factor. This leads to expressing exponential relationships using functions of the form $f(x) = a \cdot b^x$, where a is the value of the function when $x = 0$ and b is the growth factor.

Students use different rational inputs, including negative values and values between integers, to better understand exponential functions and their meaning in various contexts. This includes an exploration of growth factors over intervals of different lengths. For example, the same population growth can be described using a growth factor per decade or a different growth factor per year.

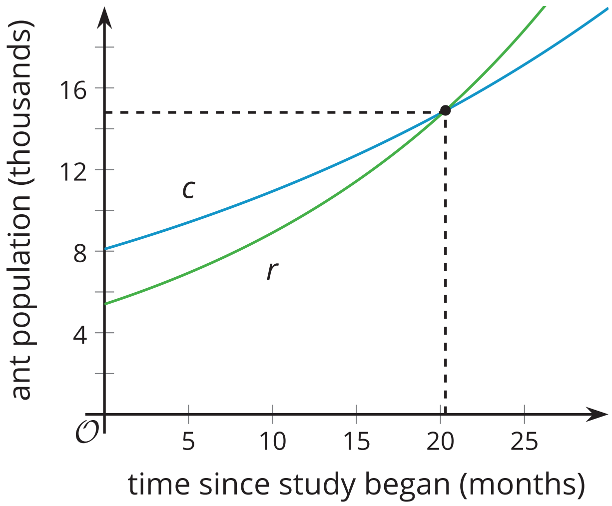
The exploration of variables used as an exponent leads to a need to solve equations for the variable and to the introduction of logarithms. Students are presented with traditional logarithm patterns and asked to find patterns and relationships, which leads them to discover that logarithms represent a way to rewrite exponential equations. That is, $a^y = x$ can be rewritten as $\log_a(x) = y$.

The constant e is introduced and students compare functions of the form $f(t) = P \cdot e^{rt}$ with functions of the form $g(t) = P \cdot (1 + r)^t$.

Then students explore logarithm rules including the product, quotient, and power rules as well as the change of base rule. The rules continue to reinforce that $\log_{10}(x) = \log(x)$ and $\log_e(x) = \ln(x)$, and the rules provide a way to use technology and the change of base rule to approximate other logarithms .

Finally, students solve exponential and logarithmic equations using graphical and algebraic methods and then interpret the solutions in context. The logarithms in this unit are primarily focused on the bases 10, 2, and e, although other positive bases are mentioned.

Note that, throughout the unit, the bases for both exponential expressions and logarithms are assumed to be positive.



Section A: Growing and Shrinking

- Lesson 1: Growing and Shrinking
- Lesson 2: Representations of Growth and Decay
- Lesson 3: Understanding Rational Inputs
- Lesson 4: Representing Functions at Rational Inputs
- Lesson 5: Changes Over Rational Intervals
- Lesson 6: Writing Equations for Exponential Functions
- Lesson 7: Interpreting and Using Exponential Functions

Section B: Missing Exponents

- Lesson 8: Unknown Exponents
- Lesson 9: What Is a Logarithm?
- Lesson 10: Interpreting and Writing Logarithmic Equations
- Lesson 11: Evaluating Logarithmic Expressions

Section C: The Constant e

- Lesson 12: The Number e
- Lesson 13: Exponential Functions with Base e
- Lesson 14: Solving Exponential Equations

Section D: Logarithm Rules

- Lesson 15: Logarithm Product Rule
- Lesson 16: Logarithm Quotient Rule
- Lesson 17: Logarithm Power Rule
- Lesson 18: Logarithm Change of Base Rule
- Lesson 19: Using Logarithm Rules

Section E: Logarithmic Functions and Graphs

- Lesson 20: Using Graphs and Logarithms to Solve Problems (Part 1)
- Lesson 21: Using Graphs and Logarithms to Solve Problems (Part 2)
- Lesson 22: Logarithmic Functions

Section F: Let's Put It to Work

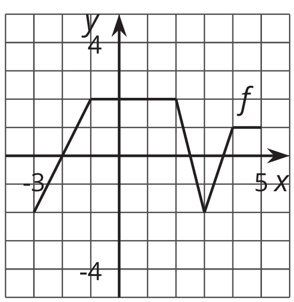
- Lesson 23: Applications of Logarithmic Functions

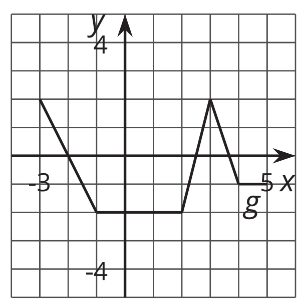
Unit 6: Transformations of Functions

In this unit, students consider functions as a whole and understand how they can be transformed to fit the needs of a situation, which is an aspect of modeling with mathematics. Prior to this unit, students have worked with a variety of function types, such as polynomial, radical, and exponential. Students will build on this work in a later unit to transform periodic functions. By saving the introduction of trigonometric functions until after a study of transformations, students have the opportunity to revisit transformations from a new perspective, which reinforces the idea that all functions, even periodic ones, behave the same way with respect to translations, reflections, and scale factors.

The unit begins with students informally describing transformations of graphs, eliciting their prior knowledge, and establishing language that will be refined throughout the unit. Students begin their investigation with vertical and horizontal translations and reflections across the axes. This leads to both algebraic and geometric descriptions of a function as even, odd, or neither.

Students continue to deepen their understanding by exploring the effects of multiplying the input and output of a function by a scale factor. Students then apply their understanding of translations, reflections, and scaling to graphs and equations of many function types. The unit ends with students applying transformations to different functions to model a real-world data set.





Section A: Translations, Reflections, and Symmetry

- Lesson 1: Matching Up to Data
- Lesson 2: Moving Functions
- Lesson 3: More Movement
- Lesson 4: Reflecting Functions
- Lesson 5: Some Functions Have Symmetry
- Lesson 6: Symmetry in Equations
- Lesson 7: Expressing Transformations of Functions Algebraically

Section B: Scaling Outputs and Inputs

- Lesson 8: Scaling the Outputs

- Lesson 9: Scaling the Inputs
- Lesson 10: Combining Functions

Section C: Transformations of Functions

- Lesson 11: Transforming from an Original Function
- Lesson 12: Transformation Effects
- Lesson 13: Transforming Parabolas
- Lesson 14: Transforming Circles

Section D: Let's Put It to Work

- Lesson 15: Making a Model for Data

Unit 7: Trigonometric Functions

In this unit, students are introduced to trigonometric functions. This unit builds directly on the work of a previous unit by having students apply their knowledge of transformations to trigonometric functions and use these functions to model periodic situations.

The unit begins with a deep study of the unit circle, as students study circular motion within familiar contexts. Students then use the Pythagorean Theorem and right-triangle trigonometry to determine the coordinates of points on a circle. They use the similarity of circles and right triangles to generalize to the unit circle, focusing on the important fact that when the hypotenuse has unit length, the length of the legs can be expressed with cosine and sine.

Then students transition to thinking about cosine and sine as functions. They use the unit circle to graph cosine and sine, then expand the domain to all real numbers as they learn the meaning of radian angles greater than 2π and less than 0. Students also reason about the input values where the tangent function does not exist and how the output values repeat at regular intervals, leading to the tangent function's periodic nature.

Next, students apply their previous work with transformations of graphs to trigonometric functions as they identify important features of periodic functions—including midline, amplitude, and period. They apply the work of this unit by modeling periodic or approximately periodic situations both algebraically and graphically.

Students create their own unit circle display in the unit. This display is meant to be a reference tool for students throughout the unit as they transition from a right-triangle-focused understanding of trigonometry to seeing cosine, sine, and tangent as functions with their own inputs and outputs. Students should have access to a unit circle display throughout the unit unless otherwise noted.

Section A: The Unit Circle

- Lesson 1: Moving in Circles
- Lesson 2: Revisiting Right Triangles
- Lesson 3: The Unit Circle (Part 1)
- Lesson 4: The Unit Circle (Part 2)
- Lesson 5: The Pythagorean Identity (Part 1)
- Lesson 6: The Pythagorean Identity (Part 2)
- Lesson 7: Finding Unknown Coordinates on a Circle

Section B: Periodic Functions

- Lesson 8: Rising and Falling
- Lesson 9: Introduction to Trigonometric Functions
- Lesson 10: Beyond 2π
- Lesson 11: Extending the Domain of Trigonometric Functions
- Lesson 12: Tangent
- Lesson 13: Some New Ratios

Section C: Trigonometry Transformations

- Lesson 14: Amplitude and Midline
- Lesson 15: Transforming Trigonometric Functions
- Lesson 16: Features of Trigonometric Graphs (Part 1)
- Lesson 17: Features of Trigonometric Graphs (Part 2)
- Lesson 18: Comparing Transformations
- Lesson 19: Modeling Circular Motion

Section D: Let’s Put It to Work

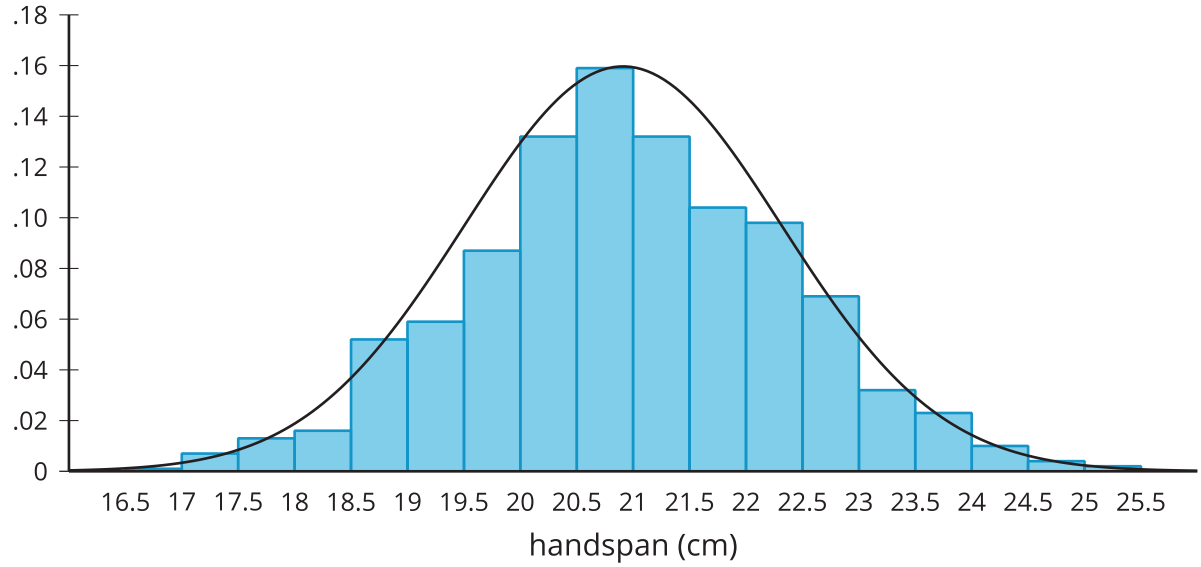
- Lesson 20: Beyond Circles

Unit 8: Statistical Inferences

This unit focuses on some important uses of randomization in statistics. Initially, students consider three types of study (experimental, observational, and survey) as ways of collecting data. In each form of study, random selection of participants and assignment into any subgroups is important for being able to generalize findings to a larger population. Students notice that an estimation of a proportion or mean of a population from sample data comes with some variability because different samples can result in different estimates. This can be expressed by including a margin of error along with any point estimates given.

Next, students examine the normal distribution as a common model for bell-shaped distributions. With technology, students can use a normal distribution model to approximate the proportion of data within certain intervals. This provides a way to quantify the confidence students should have in their estimates of population proportions or means. It also provides a way to check whether data from an experimental study has enough evidence to conclude that a difference in means between a control and treatment group is due to the treatment.

The unit concludes with an experimental study that students can do together, from study design to data collection and analysis. Then students draw a conclusion about the experiment based on their analysis.



Section A: Study Types

- Lesson 1: Being Skeptical
- Lesson 2: Study Types
- Lesson 3: Randomness in Groups

Section B: Distributions

- Lesson 4: Describing Distributions
- Lesson 5: Normal Distributions
- Lesson 6: Areas in Histograms
- Lesson 7: Areas under a Normal Curve

Section C: Not All Samples Are the Same

- Lesson 8: Not Always Ideal
- Lesson 9: Variability in Samples
- Lesson 10: Estimating Proportions from Samples
- Lesson 11: Estimating a Population Mean

Section D: Analyzing Experimental Data

- Lesson 12: Experimenting
- Lesson 13: Using Normal Distributions for Experiment Analysis
- Lesson 14: Questioning Experimenting

Section E: Let's Put It to Work

- Lesson 15: Heart Rates

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.
Time for modeling prompts is not included.

week 1, week 2, week 3, week 4, week 5, week 6, week 7, week 8, week 9, week 10, week 11, week 12, week 13, week 14, week 15, week 16, week 17, week 18, week 19, week 20, week 21, week 22, week 23, week 24, week 25, week 26, week 27, week 28, week 29, week 30, week 31, week 32

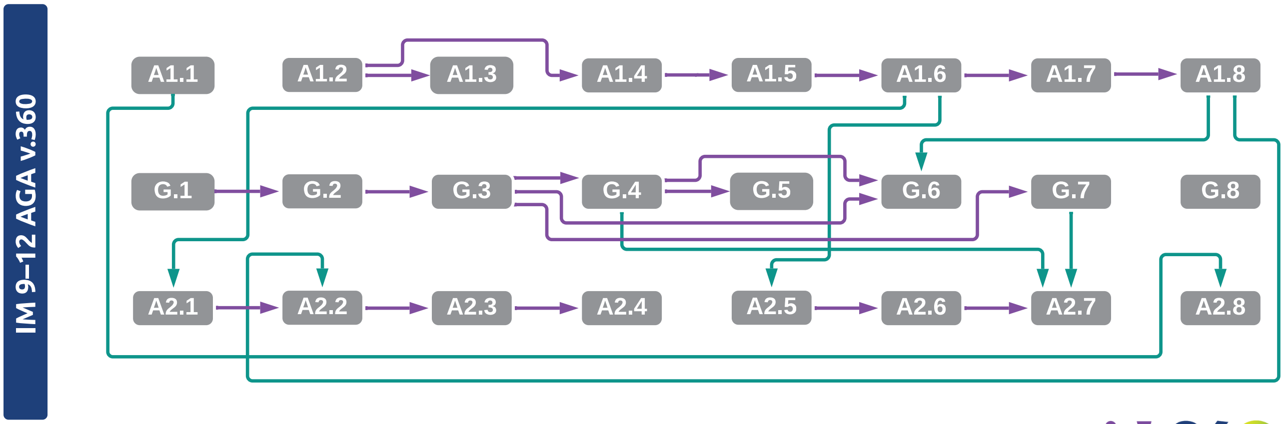
Algebra 1	Geometry	Algebra 2
Unit 1 One-variable Statistics 13–18 days Optional Lessons: 2, 5, 6, 7, 8	Unit 1 (MA) Constructions and Rigid Transformations 22–25 days Optional Lessons: 8, 18, 22	Unit 1 Sequences and Functions 12–13 days Optional Lesson: 4
Unit 2 Linear Equations and Systems 16–21 days Optional Lessons: 2, 4, 5, 18, 19	Unit 2 Congruence 16–17 days Optional Lesson: 11	Unit 2 Polynomials 17 days Optional Lessons: none
Unit 3 Two-variable Statistics 11–12 days Optional Lesson: 10	Unit 3 Similarity 16–19 days Optional Lessons: 2, 10, 12	Unit 3 Rational Functions & Identities 13 days Optional Lessons: none
Unit 4 Linear Inequalities and Systems 11 days Optional Lessons: none	Unit 4 Right Triangle Trigonometry 14 days Optional Lessons: none	Unit 4 Complex Nums & Rat Exponents 15–22 days Optional Lessons: 1, 2, 9, 14, 16, 19, 20
Unit 5 (MA) Functions 23 days Optional Lessons: none	Unit 5 Solid Geometry 20 days Optional Lessons: none	Unit 5 (MA) Exponential Functions and Equations 21–24 days Optional Lessons: 2, 7, 23
Unit 6 (MA) Introduction to Exponential Functions 24–27 days Optional Lesson: 8, 16, 17	Unit 6 Coordinate Geometry 20 days Optional Lessons: none	Unit 6 Transformations of Functions 17 days Optional Lessons: none
Unit 7 (MA) Introduction to Quadratic Functions 17–20 days Optional Lesson: 13, 14, 16	Unit 7 Circles 14 days Optional Lessons: none	Unit 7 (MA) Trigonometric Functions 23 days Optional Lessons: none
Unit 8 (MA) Quadratic Equations 26–27 days Optional Lessons: 18	Unit 8 Conditional Probability 11–13 days Optional Lessons: 1, 11	Unit 8 (MA) Statistical Inferences 17–18 days Optional Lesson: 4

(MA) = Unit has Mid-Unit Assessment

Total number of days = Lessons + Assessments – Optional Lessons

Algebra 1 = 141, Geometry = 135, Algebra 2 = 135

Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. For example, there is an arrow from A1.6 to A1.7, because when quadratic functions are introduced, they are contrasted with exponential functions, assuming that students are already familiar with exponential functions.

The following chart shows unit dependencies between 6–8 and Algebra 1.