Unit 2, Lesson 1

# Let's Make a Box

Let's investigate volumes of different boxes.

## 1.1 Which Three Go Together: Boxes

Which three go together? Why do they go together?

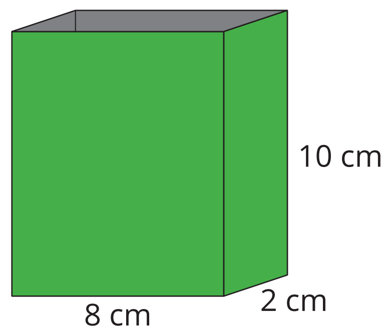
**A**

length: 4cm

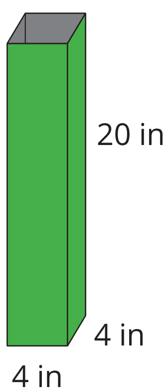
width: 8cm

height: 10cm

**B**

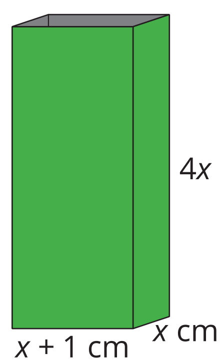



**C**




**D**

$V = 320 \text{ cm}^3$



## 1.2 Building Boxes

Your teacher will give you some supplies to construct an open-top box.

1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
2. Calculate the volume of your box, and complete the table with your information.

| side length of square cutout (in) | length (in) | width (in) | height (in) | volume of box ($in^3$) |
|---|---|---|---|---|
| 1 | | | | |
| | | | | |
| | | | | |

## 1.3 Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length $x$ in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

   Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

3. Use graphing technology to create a graph representing $V(x)$. Approximate the value of $x$ that would allow you to construct an open-top box with the largest volume possible from one piece of paper.

## Are you ready for more?

The surface area $A(x)$, in square inches, of the open-top box is also a function of the side length $x$, in inches, of the square cutouts.

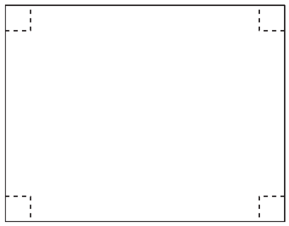
1. Find one expression for $A(x)$ by adding the area of the five faces of our open-top box.

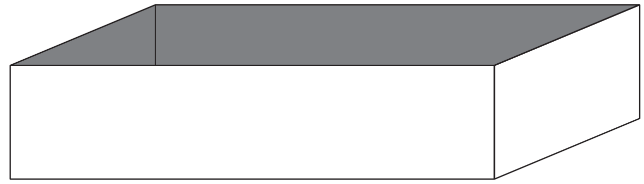
2. Find another expression for $A(x)$ by subtracting the area of the cutouts from the area of the paper.

3. Show algebraically that these two expressions are equivalent.

## Lesson 1 Summary

A box can be created by removing squares from each corner of a rectangle of paper.





Let $V(x)$ be the volume of the box in cubic inches, where $x$ is the side length, in inches, of each square removed from the four corners.

To define $V$ using an expression, we can use the fact that the volume of a cube is $(length)(width)(height)$. If the piece of paper we start with is 3 inches by 8 inches, then:

$$V(x) = (3 - 2x)(8 - 2x)(x)$$

What are some reasonable values for $x$? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since $3 - 1.5 \cdot 2 = 0$). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of $y = V(x)$ for $x$ values between 0 and 1.5. It looks like the largest volume we can get for a box made this way from a 3-inch by 8-inch piece of paper is about 7.4 $in^3$.