



# Angle-Side-Angle Triangle Congruence

Let's see if we can prove other sets of measurements that guarantee triangles are congruent, and apply those theorems.

## 7.1 Notice and Wonder: Assertion

Assertion: Through 2 distinct points passes a unique line. Two lines are said to be *distinct* if there is at least 1 point that belongs to one but not the other. Otherwise, we say the lines are the same. Lines that have no point in common are said to be *parallel*.

Conclusion: Given two distinct lines, either they are parallel, or they have exactly 1 point in common.

What do you notice? What do you wonder?

## 7.2 Proving the Angle-Side-Angle Triangle Congruence Theorem

1. Two triangles have 2 pairs of corresponding angles congruent, and the corresponding sides between those angles are congruent. Sketch two triangles that fit this description.
2. Label the triangles  $WXY$  and  $DEF$  so that angle  $W$  is congruent to angle  $D$ , angle  $X$  is congruent to angle  $E$ , and side  $WX$  is congruent to side  $DE$ .

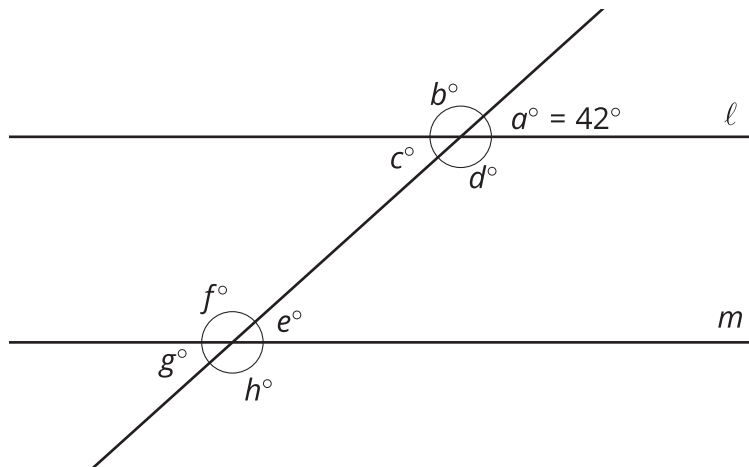


3. Use a sequence of rigid motions to take triangle  $WXY$  onto triangle  $DEF$ . For each step, explain how you know that one or more vertices will line up.

### 7.3 Find the Unknown Angle Measures

Lines  $\ell$  and  $m$  are parallel.  $a = 42$ . Find  $b$ ,  $c$ ,  $d$ ,  $e$ ,  $f$ ,  $g$ , and  $h$ .

$\ell \parallel m$



Quadrilateral  $ABCD$  is a parallelogram. By definition, that means that segment  $AB$  is parallel to segment  $CD$ , and segment  $BC$  is parallel to segment  $AD$ .

1. Sketch parallelogram  $ABCD$  and then draw an auxiliary line to show how  $ABCD$  can be decomposed into 2 triangles.
2. Prove that the 2 triangles you created are congruent.
3. Write a 1–2 sentence summary of why one pair of opposite sides of a parallelogram must be congruent.



### Are you ready for more?

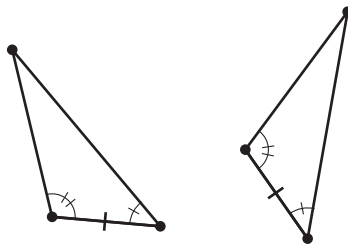
When we have 3 consecutive vertices of a polygon  $A$ ,  $B$ , and  $C$  so that the triangle  $ABC$  lies entirely inside the polygon, we call  $B$  an *ear* of the polygon.

1. How many ears does a parallelogram have?
2. Draw a quadrilateral that has fewer ears than a parallelogram.
3. In 1975, American mathematician Gary Meisters proved that every polygon has at least 2 ears. Draw a hexagon with only 2 ears.

## Lesson 7 Summary

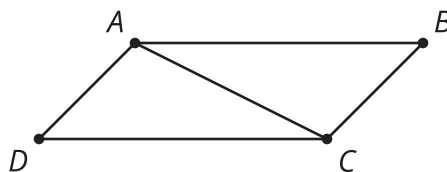
We know that in two triangles, if 2 pairs of corresponding sides and the pair of corresponding angles between the sides are congruent, then the triangles must be congruent. But we don't always know that 2 pairs of corresponding sides are congruent. For example, when proving that opposite sides are congruent in any parallelogram, we only have information about 1 pair of corresponding sides. That is why we need ways other than the Side-Angle-Side Triangle Congruence Theorem to prove triangles are congruent.

In two triangles, if 2 pairs of corresponding angles and the pair of corresponding sides between the angles are congruent, then the triangles must be congruent. This is called the *Angle-Side-Angle Triangle Congruence Theorem*.



When proving that two triangles are congruent, look at the diagram and given information, and think about whether it will be easier to find 2 pairs of corresponding angles that are congruent or 2 pairs of corresponding sides that are congruent. Then check if there is enough information to use the Angle-Side-Angle Triangle Congruence Theorem or the Side-Angle-Side Triangle Congruence Theorem.

The Angle-Side-Angle Triangle Congruence Theorem can be used to prove that, in a parallelogram, opposite sides are congruent. A parallelogram is defined to be a quadrilateral in which the 2 pairs of opposite sides are parallel.



We could prove that triangles  $ABC$  and  $CDA$  are congruent by the Angle-Side-Angle Triangle Congruence Theorem. Then we can say segment  $AD$  is congruent to segment  $CB$  because they are corresponding parts of congruent triangles.