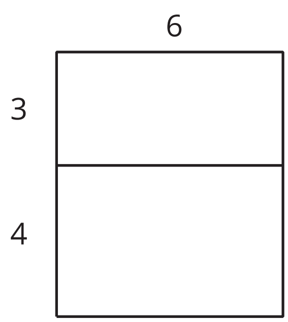
Unit 4, Lesson 8

Equivalent Quadratic Expressions

Let's use diagrams to help us rewrite quadratic expressions.

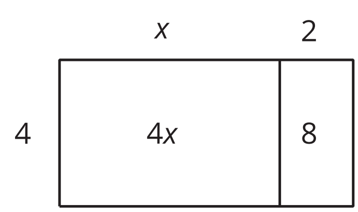
8.1 Diagrams of Products



1. Explain why the diagram shows that $6(3+4) = 6 \cdot 3 + 6 \cdot 4$.
2. Draw a diagram to show that $5(x+2) = 5x + 10$.

8.2 Drawing Diagrams to Represent More Products

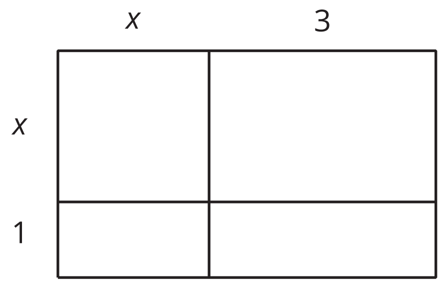
Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths of $(x+2)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n(2n+5)$ and $2n^2 + 5n$ are equivalent expressions.
2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
   a. $6\left(\frac{1}{3}n + 2\right)$
   b. $p(4p+9)$
   c. $5r\left(r + \frac{3}{5}\right)$
   d. $(0.5w + 7)w$

8.3 Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths of $x + 1$ and $x + 3$. Use this diagram to show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.

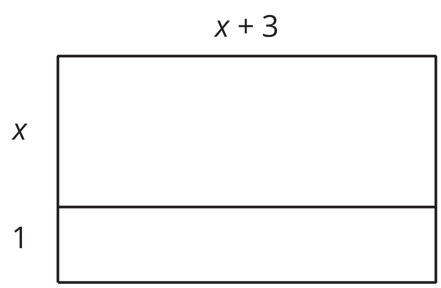


2. Draw diagrams to help you write an equivalent expression for each of the following:

   a. $(2x + 1)(x + 3)$

   b. $(x + 5)^2$

3. Here is a diagram of a rectangle with the same area as in the first question. Use this diagram to show that $(x + 1)(x + 3)$ and $x(x + 3) + 1(x + 3)$ are equivalent expressions. Then explain how you could rewrite that expression as $x^2 + 4x + 3$, without a diagram.

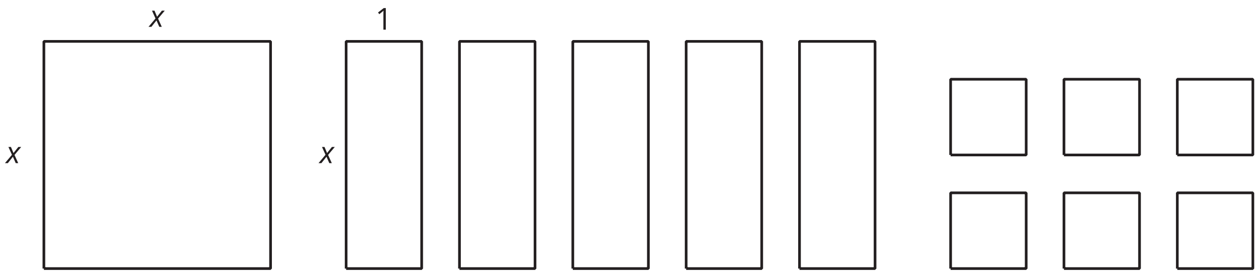


4. Write an equivalent expression for each expression:

   a. $(x + 2)(x + 6)$

   b. $(x + m)(x + n)$

Are you ready for more?

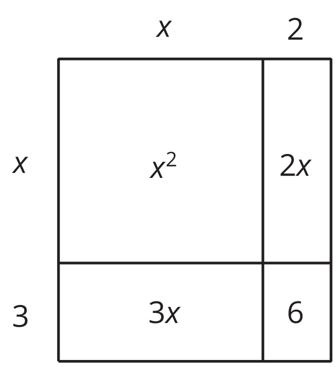


1. Is it possible to arrange an x-by-x square, five x-by-1 rectangles and six 1-by-1 squares into a single large rectangle? Explain or show your reasoning.
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?
3. Keeping the x-by-x square and the five x-by-1 rectangles, can you form a different rectangle by using a different number of 1-by-1 squares than what is shown?

Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed with $x(18 - x)$, which can also be written as $18x - x^2$.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x + 2)(x + 3)$. We can write an equivalent expression by thinking about each factor, the $(x + 2)$ and $(x + 3)$, as the side lengths of a rectangle, with each side length being decomposed into a variable expression and a number.



Multiplying $(x + 2)$ and $(x + 3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x + 2)(x + 3)$ is equivalent to

$x^2 + 2x + 3x + 6$, or to $x^2 + 5x + 6$.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x + 2$) is multiplied by every term in the other factor (the x and the 3 in $x + 3$).

$$
\begin{aligned}
&(x + 2)(x + 3) \\
&= x(x + 3) + 2(x + 3) \\
&= x^2 + 3x + 2x + (2)(3) \\
&= x^2 + (3 + 2)x + (2)(3)
\end{aligned}
$$

In general, when a quadratic expression is written in the form of $(x + p)(x + q)$, we can apply the distributive property to rewrite it as $x^2 + px + qx + pq$, or as $x^2 + (p + q)x + pq$.