Inverse Functions

Let's define functions forward and backward.

17.1 What Does It Say?

Here is an *encoded* message, a message that has been converted into a code.

WRGDB LV D JRRG GDB.

Can you figure out what it says in English? How was the original message encoded?

17.2 Caesar Says, "Shift"

1. Now it's your turn to write a secret code!
    a. Write a short and friendly message with 3–4 words.

    b. Pick a number from 1 to 10. Then, encode your message by shifting each letter that many steps forward or backward in the alphabet, wrapping around from *Z* to *A* as needed.

    Complete these tables to create a key for your cipher.

position in the alphabet	1	2	3	4	5	6	7	8	9	10	11	12	13
letter in the message	A	B	C	D	E	F	G	H	I	J	K	L	M
letter in code													

position in the alphabet	14	15	16	17	18	19	20	21	22	23	24	25	26
letter in the message	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
letter in code													

c. Give your encoded message to a partner to decode. If requested, give the number you used.

d. Decode the message from your partner. Ask for their number, if needed.

2. Each letter can be represented by a number. For example, *F* is 6 because it is the 6th letter of the alphabet.

    - Complete the first 2 rows of the table to convert between letters and numbers.
    - Complete the third row by adding or subtracting the number you chose in the last problem to find the coded letter number.
    - Complete the fourth row by converting the coded number to a letter.

letter in message	F	I	S	
message letter number m	6			8
coded letter number c				
letter in code				

3. Use m and c to write an equation that can be used to *encode* an original message into your secret code.

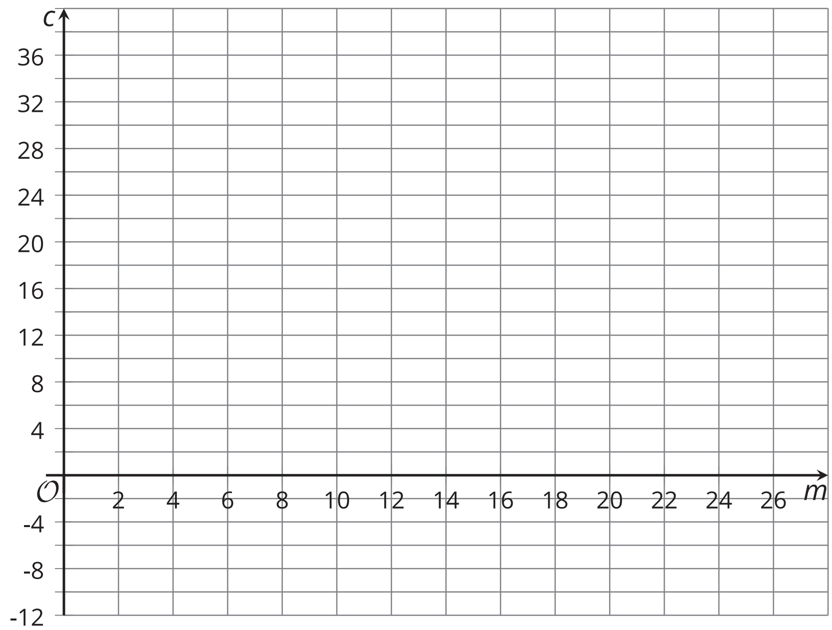
4. Use m and c to write an equation that can be used to *decode* your secret code into the original message.

Are you ready for more?

There are 26 letters in the alphabet, so only the numbers 1–26 make sense for m and c.

1. Try using the equation that you wrote to encode the letters *A*, *B*, *Y*, and *Z*. Did you end up with position numbers or c values that are less than 1 or greater than 26? For which letters?

2. Use your encoding equation to plot the (m, c) pairs for all the letters in the alphabet.



3. Look for the points whose c value is less than 1 or greater than 26. What letters should they be in the code? Plot the points where they should be according to the rule of your cipher.

4. Did you end up with a graph of a piecewise function? If so, can you describe the different rules that apply to different domains of the function?

17.3 Japanese Yen and Peruvian Soles

A Japanese traveler who is heading to Peru exchanges some Japanese yen for Peruvian soles. At the time of his travel, 1 yen can be exchanged for 39.77 soles.

At the same time, a Peruvian businesswoman who is in Japan is exchanging some Peruvian soles for Japanese yen at the same exchange rate.

1. Find the amount of money in soles that the Japanese traveler would get if he exchanged:
    a. 100 yen
    b. 500 yen
2. Write an equation that gives the amount of money in soles, s, as a function of the amount of money in yen, y, being exchanged.
3. Find the amount that the Peruvian businesswoman would get if she exchanged:
    a. 1,000 soles
    b. 5,000 soles
4. Explain why it might be helpful to write the inverse of the function you wrote earlier. Then, write an equation that defines the inverse function.

Lesson 17 Summary

Sometimes it is useful to reverse a function so that the original output is now the input.

Suppose Han lives 400 meters from school and walks to school. A linear function gives Han's distance to school, D, in meters, after he has walked w meters from home, and is defined by:

$$D = 400 - w$$

With this equation, if we know how far Han has walked from home, w, we can easily find his remaining distance to school, D. Here, w is the input, and D is the output.

What if we know Han's remaining distance to school, D, and want to know how far he has walked, w?

We can find out by solving for w:

$$\begin{aligned} D &= 400 - w \\ D + w &= 400 \\ w &= 400 - D \end{aligned}$$

The equation $w = 400 - D$ represents the *inverse* of the original function.

With this equation, we can easily find how far Han has walked from home if we know his remaining distance to school. Here, w and D have switched roles: w is now the output, and D is the input.

In general, if a function takes a as its input and gives b as its output, its **inverse function** takes b as the input and a as the output.

