Unit 4, Lesson 10

Dividing by Unit and Non-Unit Fractions

Let's look for patterns when we divide by a fraction.

10.1 Dividing by a Whole Number

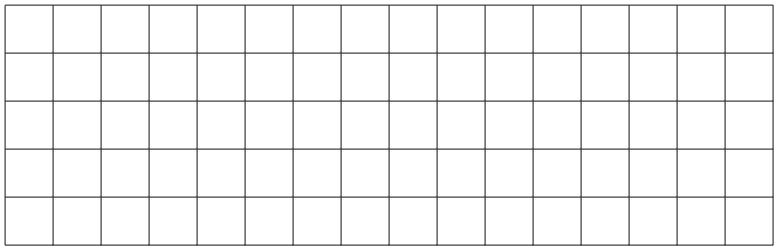
Work with a partner. One person solves the problems labeled "Partner A" and the other person solves those labeled "Partner B."

Write an equation for each question. If you get stuck, consider drawing a diagram.

1. Partner A:

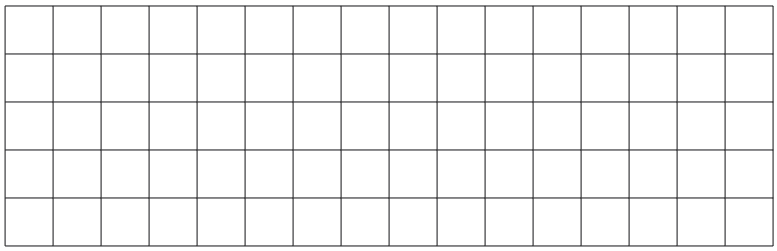
   How many 3s are in 12?

   Division equation:



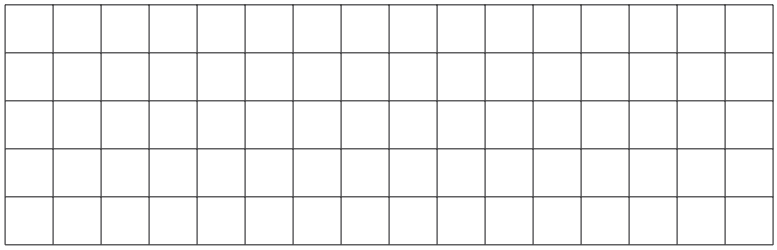
   How many 4s are in 12?

   Division equation:



   How many 6s are in 12?

   Division equation:

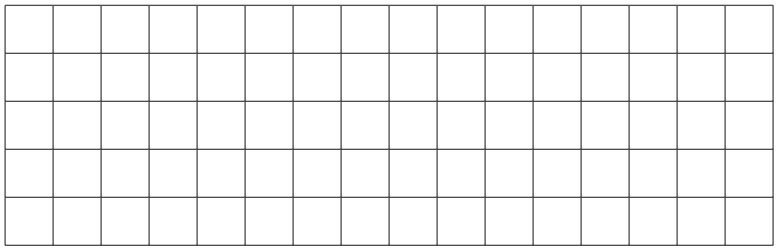


CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Partner B:

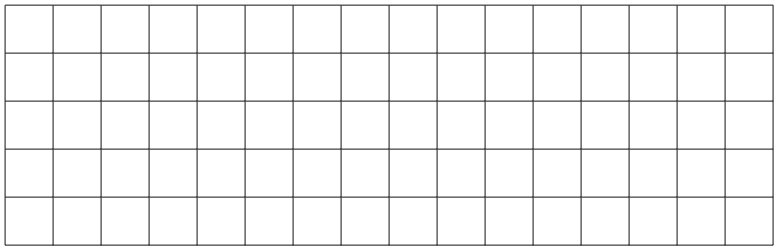
What is 12 groups of $\frac{1}{3}$?

Multiplication equation:



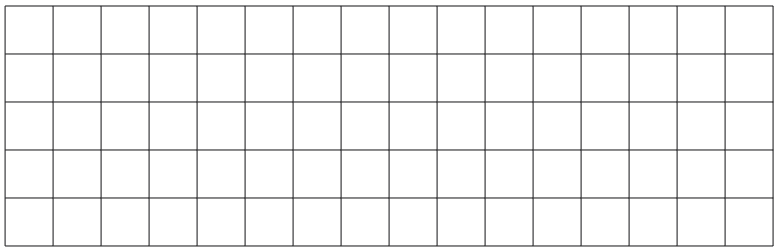
What is 12 groups of $\frac{1}{4}$?

Multiplication equation:



What is 12 groups of $\frac{1}{6}$?

Multiplication equation:



2. What do you notice about the diagrams and equations? Discuss with your partner.

3. Complete this sentence based on what you noticed:

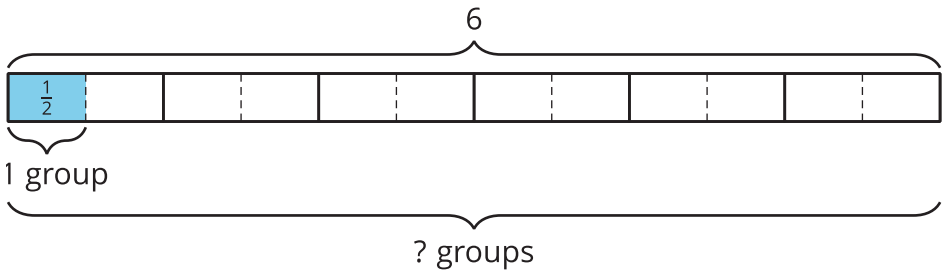
   Dividing by a whole number a produces the same result as multiplying by _____ .

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

10.2 Dividing by Unit Fractions

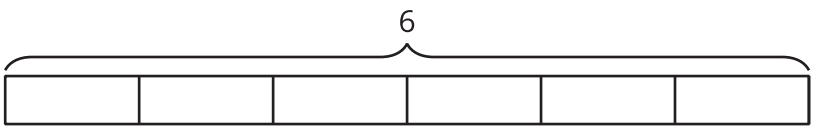
1. To find the value of $6 \div \frac{1}{2}$, Elena thought, "How many $\frac{1}{2}$s are in 6?" and then she drew this tape diagram. It shows 6 ones, with each one partitioned into 2 equal pieces.

$6 \div \frac{1}{2}$



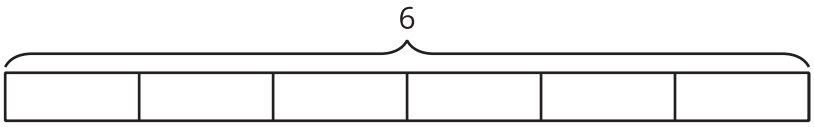
For each division expression, complete the diagram using the same method as Elena. Then, find the value of the expression.

a. $6 \div \frac{1}{3}$



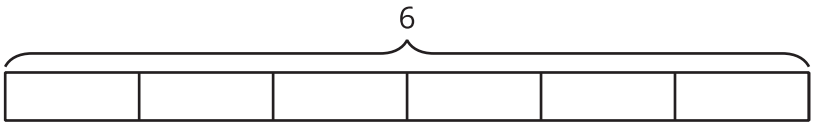
Value of the expression: _____

b. $6 \div \frac{1}{4}$



Value of the expression: _____

c. $6 \div \frac{1}{6}$



Value of the expression: _____

2. Look for a pattern in the expressions and their values. Talk to your partner about how to find how many halves, thirds, fourths, or sixths were in 6 wholes, without counting all the parts.

3. Use the pattern you noticed to find the values of these expressions. If you get stuck, consider drawing a diagram.

   a. $6 \div \frac{1}{8}$

   b. $6 \div \frac{1}{10}$

   c. $6 \div \frac{1}{25}$

   d. $6 \div \frac{1}{b}$

4. Find the value of each expression.

   a. $8 \div \frac{1}{4}$

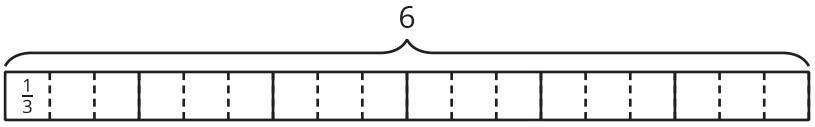
   b. $12 \div \frac{1}{5}$

   c. $a \div \frac{1}{2}$

   d. $a \div \frac{1}{b}$

10.3 Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena started by drawing this diagram.



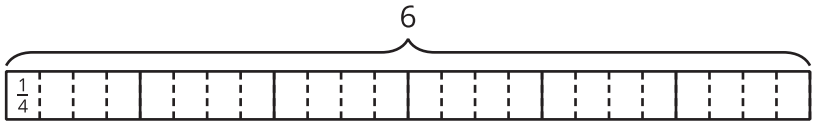
   a. Complete the diagram to show how many $\frac{2}{3}$s are in 6.

   b. Elena says, "To find $6 \div \frac{2}{3}$, I can take the value of $6 \div \frac{1}{3}$ and then either multiply it by $\frac{1}{2}$ or divide it by 2."

      Discuss with your partner why Elena's method works.

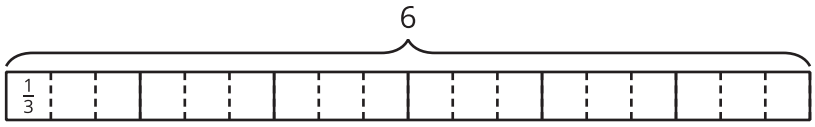
2. Use the diagram and Elena's method to find the value of each expression. Think about how to find that value without counting all the pieces in the diagram.

   a. $6 \div \frac{3}{4}$



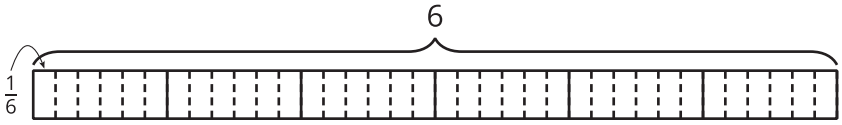
   Value of the expression: _____

   b. $6 \div \frac{4}{3}$



   Value of the expression: _____

   c. $6 \div \frac{4}{6}$



   Value of the expression: _____

3. Elena noticed that she always took the same two steps to show division by a fraction on a tape diagram. She said:

   "First, I would partition each 1 whole into as many parts as the number in the denominator. For $6 \div \frac{3}{4}$, that number is 4, so the diagram would have 4 times as many parts.

   Next, I would put a certain number of those parts into one group. For $6 \div \frac{3}{4}$, I would put 3 of the $\frac{1}{4}$s into each group and see how many groups there are."

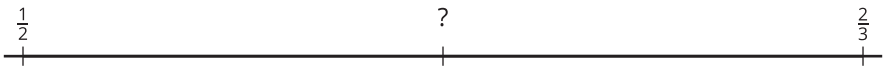
   Which expression represents the result of taking these two steps to find $6 \div \frac{3}{4}$?

   Be prepared to explain your reasoning.

   - $6 \div 4 \cdot 3$
   - $6 \cdot 4 \div 3$
   - $6 \div 4 \div 3$
   - $6 \cdot 4 \cdot 3$

Are you ready for more?

Find the unknown value.

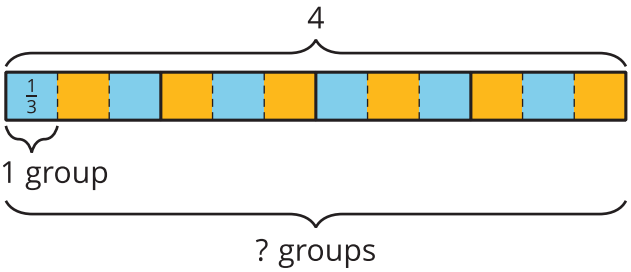


CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Lesson 10 Summary

To answer the question "How many $\frac{1}{3}$s are in 4?" or "What is $4 \div \frac{1}{3}$?", we can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4.

In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

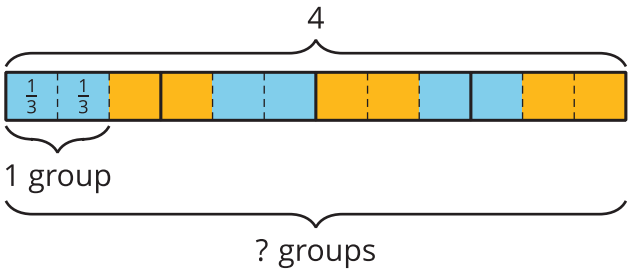


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b.

How can we reason about $4 \div \frac{2}{3}$?

We already know that there are $(4 \cdot 3)$ or 12 groups of $\frac{1}{3}$s in 4. To find how many $\frac{2}{3}$s are in 4, we need to put together every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words,



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number by b and then by $\frac{1}{a}$.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.