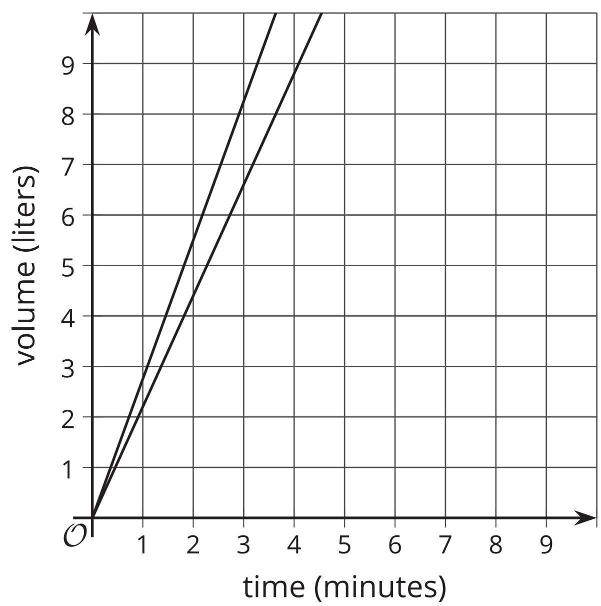
Unit 5 Family Support Materials

Linear Relationships

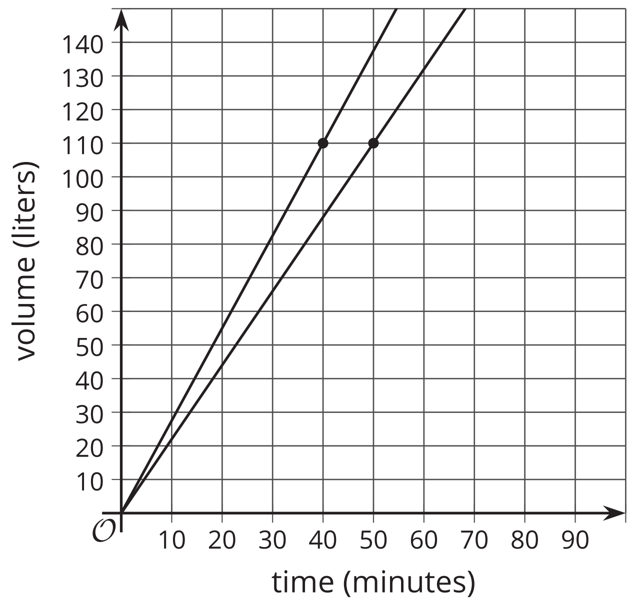
Section A: Proportional Relationships

This week your student will consider what makes a graph useful and then use graphs, equations, tables, and descriptions to compare two different situations. There are many successful ways to set up and add a scale to a pair of axes when creating a graph of a situation. Sometimes we choose specific ranges for the axes in order to see specific information.

For example, if two large, cylindrical water tanks are being filled at a constant rate, we could show the amount of water in them using a graph like this:



While the first graph is accurate, it only shows up to 10 liters, which isn't that much water. Let's say we wanted to know how long it would take each tank to have 110 liters. With 110 as a guide, we could set up our axes like this:



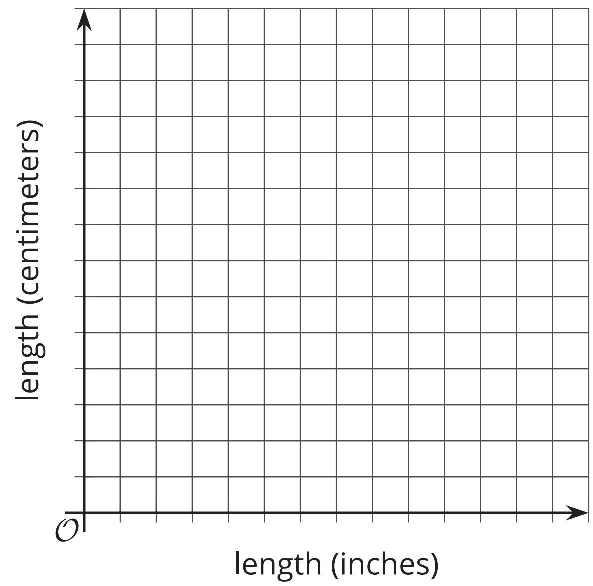
Notice how the vertical scale goes beyond the value we are interested in. Also notice how each axis has values that increase by 10, which, along with numbers like 1, 2, 5, and 25, is a friendly number to count by.

Here is a task to try with your student:

1. This table shows some lengths measured in inches and the equivalent length in centimeters. Complete the table.

length (inches)	length (centimeters)
1	2.54
2	
10	
	50.8

2. Sketch a graph of the relationships between inches and centimeters. Scale the axes so that all the values in the table can be seen on the graph.

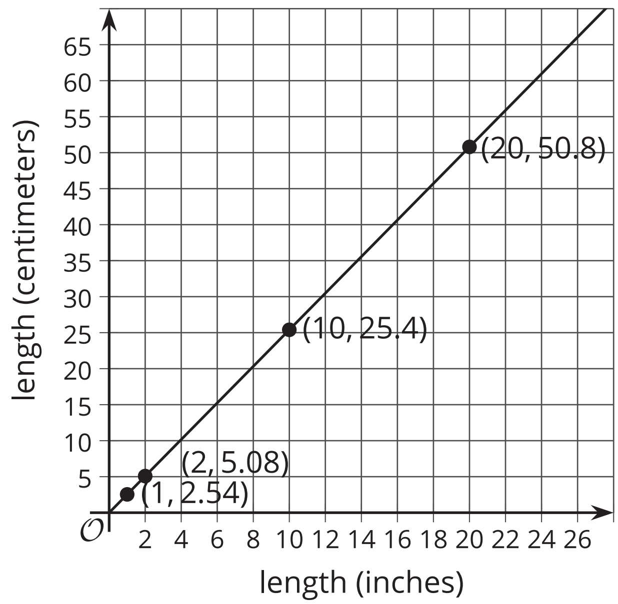


Solution:

1.

length (inches)	length (centimeters)
1	2.54
2	5.08
10	25.4
20	50.8

2.


length (centimeters)
65
60
55
50
45
40
35
30
25
20
15
10
5
(20, 50.8)
(10, 25.4)
(2, 5.08)
(1, 2.54)
O
2 4 6 8 10 12 14 16 18 20 22 24 26
length (inches)

Section B: Representing Linear Relationships

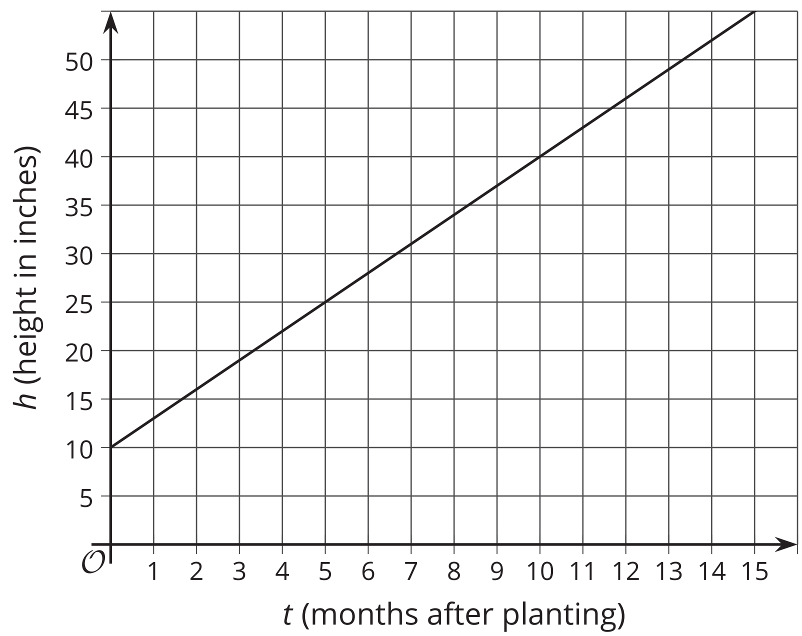
This week your student will learn how to write equations representing **linear relationships**. A linear relationship exists between two quantities when one quantity has a constant rate of change with respect to the other. The relationship is called linear because its graph is a line.

For example, say we are 5 miles into a hike heading toward a lake at the end of the trail. If we walk at a speed of 2.5 miles per hour, then for each hour that passes we are 2.5 miles further along the trail. After 1 hour, we would be 7.5 miles from the start. After 2 hours, we would be 10 miles from the start (assuming no stops). This means there is a linear relationship between miles traveled and hours walked. A graph representing this situation is a line with a slope of 2.5 and a **vertical intercept** of 5.

Here is a task to try with your student:

The graph shows the height in inches, h, of a bamboo plant t months after it has been planted.

1. What is the slope of this line? What does that value mean in this context?
2. At what point does the line intersect the vertical axis? What does that value mean in this context?

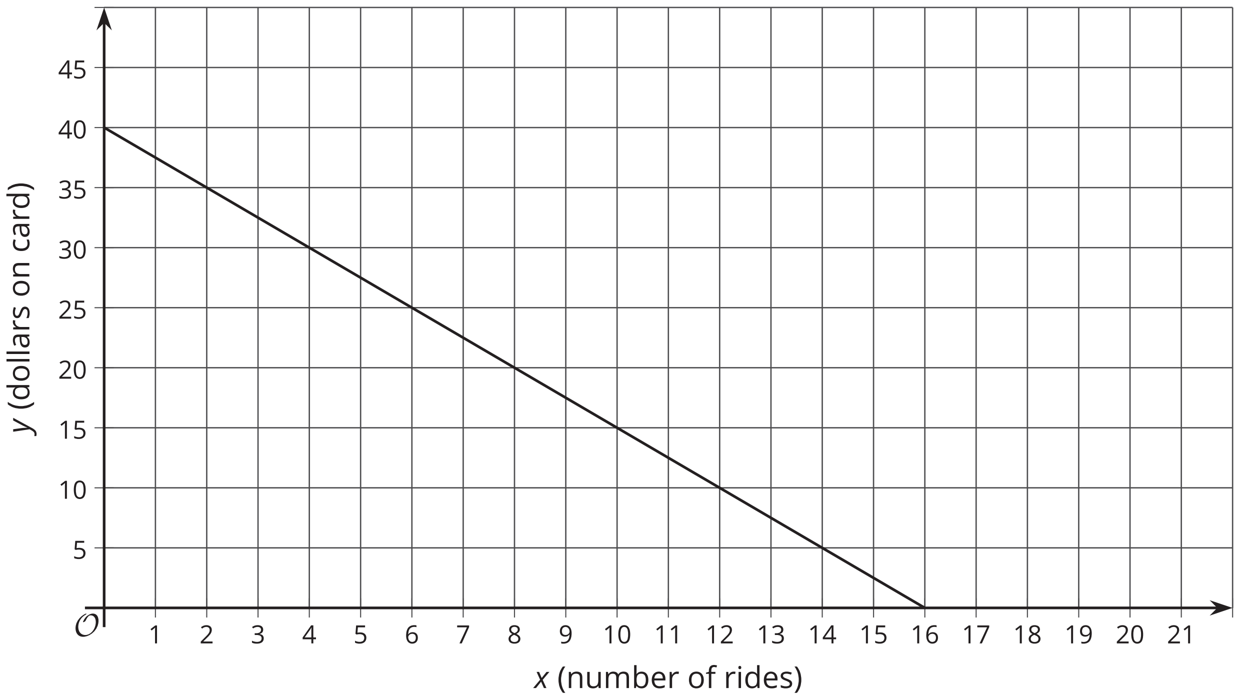


Solution:

1. The slope of the line is 3. This means that with every month that passes, the bamboo plant grows an additional 3 inches.
2. The line intersects the vertical axis at $(0, 10)$. This bamboo plant was planted when it was 10 inches tall.

Section C: Finding Slopes and Linear Equations

This week your student will investigate linear relationships with slopes that are not positive. Here is an example of a line with negative slope that represents the amount of money on a public transit fare card based on the number of rides taken:

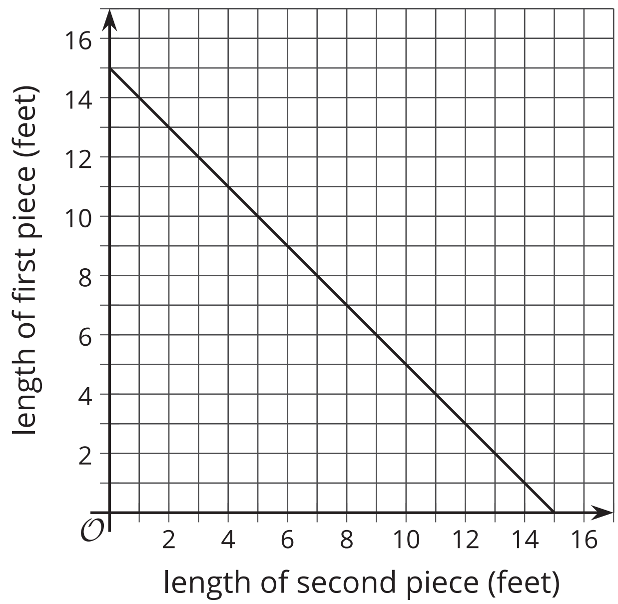


The slope of the line graphed here is -2.5, since $\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-40}{16} = -2.5$, and corresponds to the cost of 1 ride. The vertical intercept is 40, which means the card started out with $40 on it.

One possible equation for this line is $y = -2.5x + 40$. It is important to understand that every pair of numbers (x, y) that is a **solution to the equation** representing the situation is also a point on the graph representing the situation. Points *not* on the graph representing the situation will *not* be a solution to the equation representing the situation.

Here is a task to try with your student:

A length of ribbon is cut into two pieces. The graph shows the length of the first piece with respect to the length of the second piece.



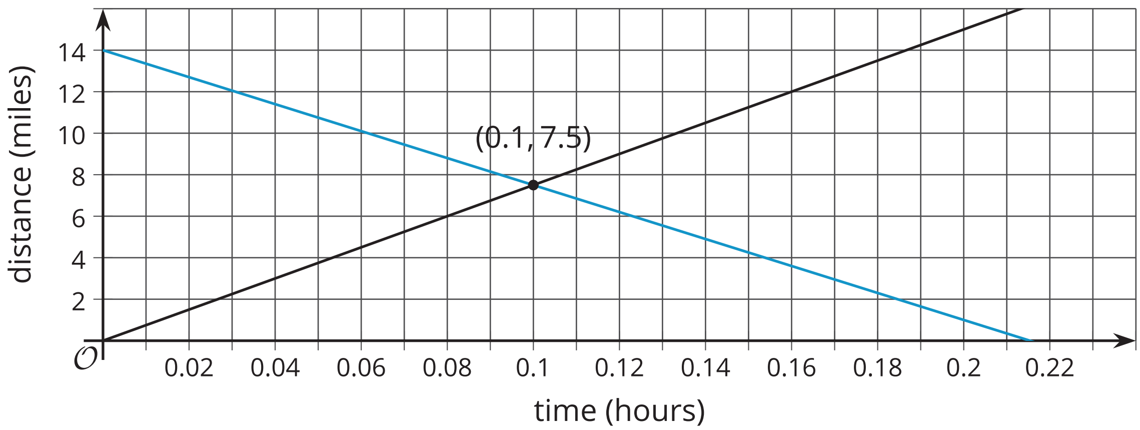
1. How long is the original ribbon? Explain how you know.
2. What is the slope of the line? What does it mean in this situation?
3. List the coordinates of three points that are on the line and explain what they mean.

Solution:

1. 15 feet. When the second piece is 0 feet long, the first is 15 feet long, so that is the length of the ribbon.
2. The slope of the line is -1. It means that for each foot the second piece increases by, the first piece must decrease by the same length. For example, if we want the second piece to be 1 foot longer, then the first piece must be 1 foot shorter.
3. Sample coordinates: The point $(14.5, 0.5)$ means the second piece is 14.5 feet long, so the first piece is only a half foot long. The point $(7.5, 7.5)$ means each piece is 7.5 feet long, so the original ribbon was cut in half. The point $(0, 15)$ means the original ribbon was not cut at all to make a second piece, so the first piece is 15 feet long.

Section D: Systems of Linear Equations

This week your student will work with systems of equations. A system of equations is a set of 2 (or more) equations where the letters represent the same values. For example, say Car A is traveling 75 miles per hour and passes a rest area. The distance in miles it has traveled from the rest area after t hours is $d = 75t$. Car B is traveling toward the rest area, and its distance from the rest area at any time is $d = 14 - 65t$. We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is "yes," then the solution will correspond to one point that is on both lines, such as the point $(0.1, 7.5)$ shown here. This means that 0.1 hours after Car A passes the rest area, both cars will be 7.5 miles from the rest area.



We could also answer the question without using a graph. Because we are asking when the d values for each car will be the same, we are asking for what t value, if any, makes $75t = 14 - 65t$ true. Solving this equation for t, we find that $t = 0.1$ is a solution, and at that time the cars are 7.5 miles away because $75t = 75 \cdot 0.1 = 7.5$. This finding matches the graph.

Here is a task to try with your student:

Lin and Diego are biking the same direction on the same path, but start at different times. Diego is riding at a constant speed of 18 miles per hour, so his distance traveled in miles can be represented by d and the time he has traveled in hours by t, where $d = 18t$. Lin started riding a quarter hour before Diego at a constant speed of 12 miles per hour, so her total distance traveled in miles can be represented by d, where $d = 12\left(t + \frac{1}{4}\right)$. When will Lin and Diego meet?

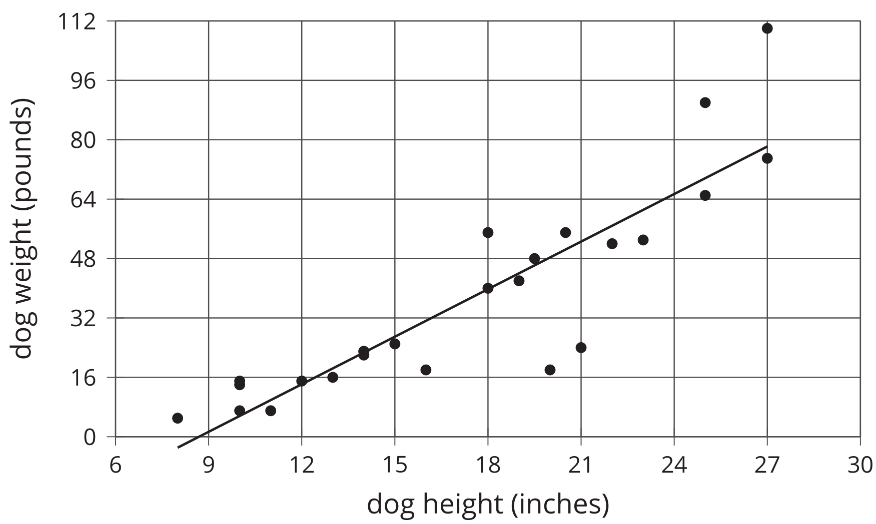
Solution:

To find when Lin and Diego meet, that is, when they have traveled the same total distance, we can set the two equations equal to one another: $18t = 12\left(t + \frac{1}{4}\right)$. Solving this equation for t, $18t = 12t + 3$, $6t = 3$, $t = \frac{1}{2}$.

They meet after Diego rides for one half hour and Lin rides for three quarters of an hour. The distance that they each travel before meeting is 9 miles, because we can use $\frac{1}{2}$ for t in Diego's equation and see that $9 = 18 \cdot \frac{1}{2}$. Another way to find a solution would be to graph both $d = 18t$ and $d = 12\left(t + \frac{1}{4}\right)$ on the same coordinate plane and interpret the point where these lines intersect.

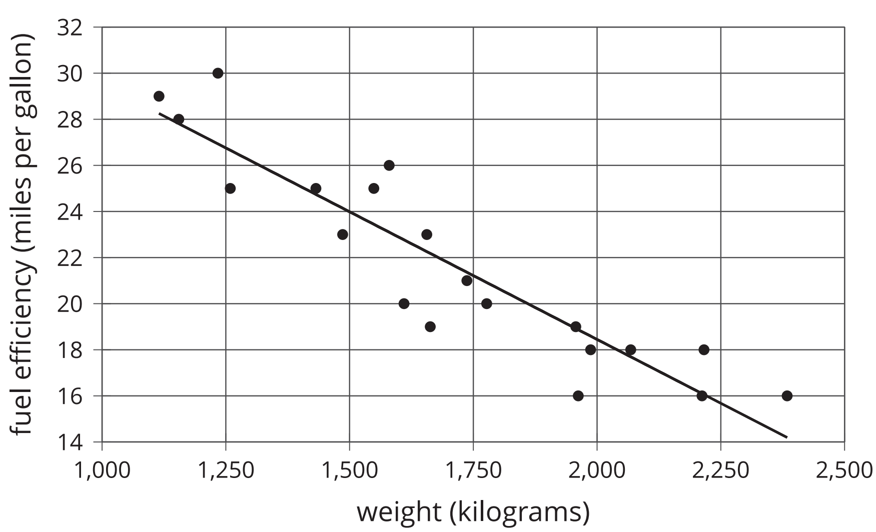
Section E: Associations in Numerical Data

This week your student will work with **scatter plots**. Scatter plots show us how two different variables are related. In this example, each plotted point corresponds to a dog, and its coordinates tell us the height and weight of that dog. The point on the lower left of the graph, for example, might represent a dog that is 8 inches tall and weighs about 5 pounds. The plot shows that, generally speaking, taller dogs weigh more than shorter dogs.



Since a larger value for one characteristic (height) generally means a larger value for the other characteristic (weight), we say that there is a **positive association** between dog height and dog weight.

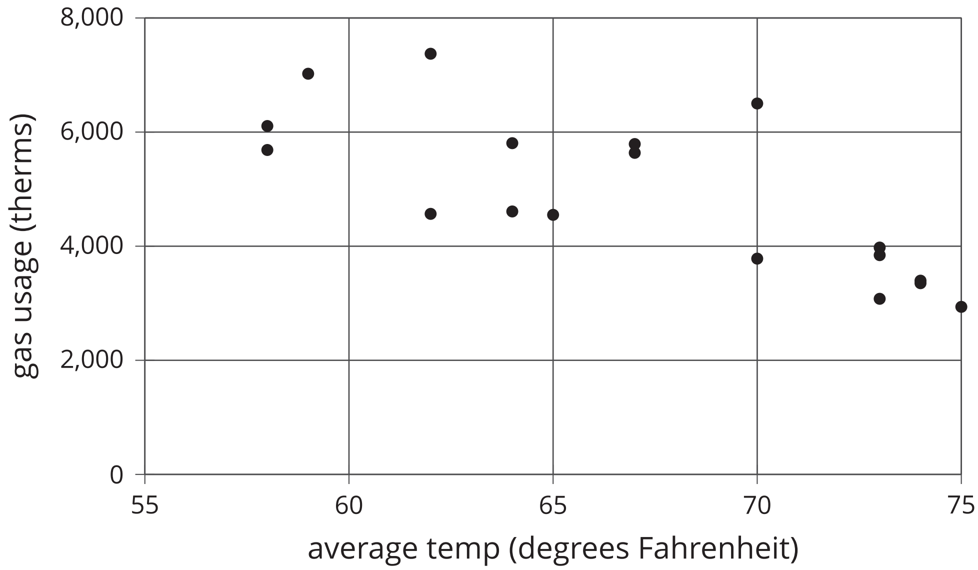
In the next example, each point corresponds to a car, and its coordinates tell us the weight (kilograms) and fuel efficiency (miles per gallon) of the car.



This time, we see that larger values for one characteristic (car weight) generally go along with lower values for the other characteristic (fuel efficiency), and so we say that there is a **negative association** between car weight and fuel efficiency.

Here is a task to try with your student:

This scatter plot shows the relationship between average temperature and gas usage in a building.



1. How many points in the graph describe the building on 70-degree days? Approximately how much gas was used on each of these days?
2. Do the variables in the gas usage for a building scatter plot show a positive association or a negative association?
3. On a 78-degree day, would the building be most likely to use (a) 1,800 therms of gas, (b) 4,200 therms of gas, or (c) 5,800 terms of gas?

Solution:

1. There are 2 points that describe gas usage for 70-degree days. On one of those days, the building used a little less than 4,000 therms of gas. On the other, the building used a little more than 6,000 therms.
2. Since less gas is used on warmer days, there is a negative association.
3. Following the trend in the graph, the building would likely use about 1,800 therms on a 78-degree day. You may draw in a line as in the dog and car scatter plots to help see this.

Section F: Associations in Categorical Data

This week your student will use **two-way tables**. Two-way tables are a way of comparing two variables. For example, this table shows the results of a study of the relation between meditation and state of mind of athletes before a track meet.

	meditated	did not meditate	total
calm	45	8	53
agitated	23	21	44
total	68	29	97

23 of the people who meditated were agitated, while 21 of the people who did not meditate were agitated. Does this mean that meditation has no impact or even a slight negative association with mood? Probably not. When we look for associations between variables, it can be more informative to know the approximate percentages in each category, like this:

	meditated	did not meditate
calm	66%	28%
agitated	34%	72%
total	100%	100%

Of the people who meditated, about 66% were calm ($45 \div 68 \cong 0.66$), and about 34% were agitated ($23 \div 68 \cong 0.34$). When we compare that to the percentages for people who did not meditate, we can now see more easily that the group of people who meditated has a lower percentage of athletes who are agitated. The percentages in this table are called **relative frequencies**.

Here is a task to try with your student:

This table contains data about whether people in various age groups use their cell phone as their main alarm clock.

	use cell phone as alarm	do not use cell phone as alarm	total
18 to 29 years old	47	16	63
30 to 49 years old	66	21	87
50+ years old	31	39	70
total	144	76	220

1. Fill in the blanks in the table below with the relative frequencies for each row. These will tell us the percentage of people in each age group who use their phone as an alarm.

	use cell phone as alarm	do not use cell phone as alarm	total
18 to 29 years old	75%, since $\frac{47}{63} \cong 0.75$		100%
30 to 49 years old			
50+ years old			

2. Comparing just the 18-to-29-year-olds and the 30-to-49-year-olds, is there an association between cell phone alarm use and age?
3. Comparing the youngest age bracket with the 50+ age bracket, is there an association between cell phone alarm use and age?

Solution:

1.

	use cell phone as alarm	do not use cell phone as alarm	total
18 to 29 years old	75%, since $\frac{47}{63} \cong 0.75$	25%, since $\frac{16}{63} \cong 0.25$	100%
30 to 49 years old	76%, since $\frac{66}{87} \cong 0.76$	24%, since $\frac{21}{87} \cong 0.24$	100%
50+ years old	44%, since $\frac{31}{70} \cong 0.44$	56%, since $\frac{39}{70} \cong 0.56$	100%

2. No. The relative frequencies are very similar.
3. Yes. Using a cell phone as an alarm is associated with being in the younger age brackets. About 75% of 18-to-29 and 30-to-49-year-olds use their cell phone as an alarm, but only about 44% of people 50 years or older do.