Unit 6, Lesson 12

Tangent

Let's learn more about tangent.

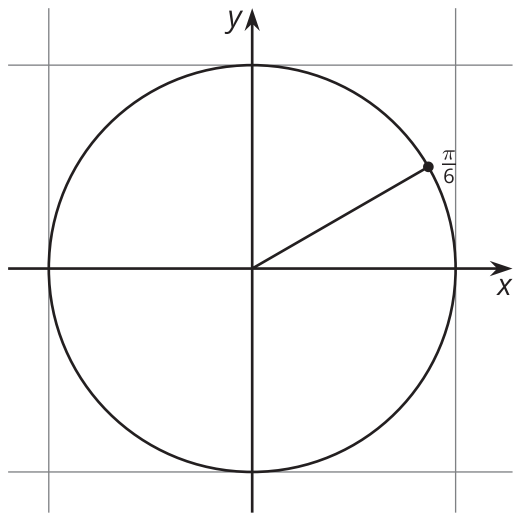
12.1 Notice and Wonder: An Unusual Function

What do you notice? What do you wonder?

θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$
$-\frac{\pi}{2}$	0	-1	
$-\frac{\pi}{3}$	0.5	-0.87	
$-\frac{\pi}{6}$	0.87	-0.5	
0	1	0	
$\frac{\pi}{6}$	0.87	0.5	
$\frac{\pi}{3}$	0.5	0.87	
$\frac{\pi}{2}$	0	1	

12.2 A Tangent Ratio

1. Complete the table. For each positive angle in the table, add the corresponding point and the segment between it and the origin to the unit circle.

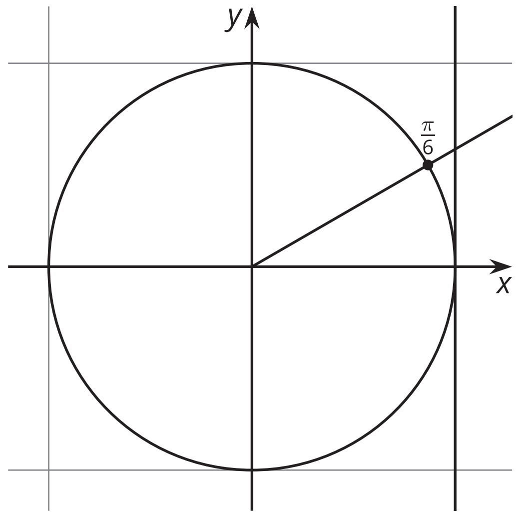


θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$
$-\frac{\pi}{2}$	0	-1	
$-\frac{\pi}{3}$	0.5	-0.87	
$-\frac{\pi}{6}$	0.87	-0.5	
0	1	0	
$\frac{\pi}{6}$	0.87	0.5	
$\frac{\pi}{3}$	0.5	0.87	
$\frac{\pi}{2}$	0	1	
$\frac{2\pi}{3}$			
$\frac{5\pi}{6}$			
π			
$\frac{7\pi}{6}$			
$\frac{4\pi}{3}$			
$\frac{3\pi}{2}$			
$\frac{5\pi}{3}$			
$\frac{11\pi}{6}$			
2π			

2. How are the values of $\tan(\theta)$ like the values of $\cos(\theta)$ and $\sin(\theta)$? How are they different?

Are you ready for more?

1. Where does the line $x = 1$ intersect the line that passes through the origin and the point corresponding to the angle $\frac{\pi}{6}$?
2. Where does the line $x = 1$ intersect the line that passes through the origin and the point corresponding to the angle θ?
3. Where do you think the name "tangent" of an angle comes from?



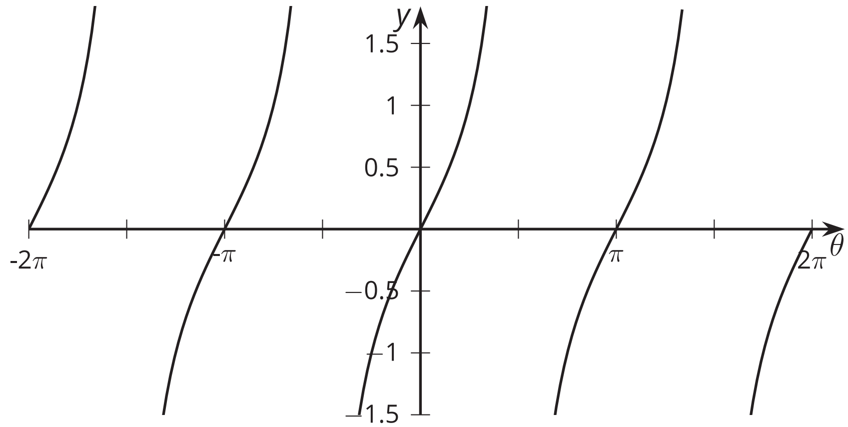
12.3 The Tangent Function

Before we graph $y = \tan(\theta)$, let's figure out some things that must be true.

1. Explain why the graph of $\tan(\theta)$ has a vertical asymptote at $x = \frac{\pi}{2}$.
2. Does the graph of $\tan(\theta)$ have other vertical asymptotes? Explain how you know.
3. For which values of θ is $\tan(\theta)$ zero? For which values of θ is $\tan(\theta)$ one? Explain how you know.
4. Is the graph of $\tan(\theta)$ periodic? Explain how you know.

Lesson 12 Summary

The tangent of an angle θ, $\tan(\theta)$, is the quotient of sine and cosine: $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. Here is a graph of $y = \tan(\theta)$.



We can see from the graph that $\tan(\theta) = 0$ when θ is $\text{-}2\pi, \text{-}\pi, 0, \pi, \text{or } 2\pi$. This makes sense because sine is 0 for these values of θ. Since sine and cosine are never 0 at the same θ, we can say that tangent has a value of 0 whenever sine has a value of 0.

We can also see the asymptotes of the tangent function: $\text{-}\frac{3\pi}{2}, \text{-}\frac{\pi}{2}, \frac{\pi}{2}, \text{and } \frac{3\pi}{2}$. Let’s look more closely at what happens when $\theta = \frac{\pi}{2}$. We have $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$. This means that $\tan\left(\frac{\pi}{2}\right) = \frac{1}{0}$, which is not defined. Whenever $\cos(\theta) = 0$, tangent is not defined and has a vertical asymptote.

Like the sine and cosine functions, the tangent function is periodic. This makes sense because it is defined using the sine and cosine functions. The period of tangent is only π, while the period of sine and cosine is 2π.

