Solving Equations with Rational Numbers

Let's solve equations that include negative values.

20.1 Math Talk: Opposites and Reciprocals

Solve each equation mentally.

- $7 \cdot b = 1$
- $c \cdot d = 1$
- $11 + f = 0$
- $g + h = 0$

20.2 Match Solutions

Match each equation to its solution.

A. $\frac{1}{2}x = -5$

B. $-2x = -9$

C. $-\frac{1}{2}x = \frac{1}{4}$

D. $-2x = 7$

E. $x + -2 = -6.5$

F. $-2 + x = \frac{1}{2}$

1. $x = -4.5$
2. $x = -\frac{1}{2}$
3. $x = -10$
4. $x = 4.5$
5. $x = 2\frac{1}{2}$
6. $x = -3.5$

Be prepared to explain your reasoning.

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20.3 Trip to the Mountains

The Hiking Club is on a trip to hike up a mountain.

1. The members increased their elevation 290 feet during their hike this morning. Now they are at an elevation of 450 feet.

   a. Explain how to find their elevation before the hike.

   b. Han says the equation $e + 290 = 450$ describes the situation. What does the variable e represent?

   c. Han says that he can rewrite his equation as $e = 450 + \text{-}290$ to solve for e. Compare Han's strategy to your strategy for finding the beginning elevation.

2. The temperature fell 4 degrees in the last hour. Now it is 21 degrees. Write and solve an equation to find the temperature it was 1 hour ago.

3. There are 3 times as many students participating in the hiking trip this year than last year. There are 42 students on the trip this year.

   a. Explain how to find the number of students that came on the hiking trip last year.

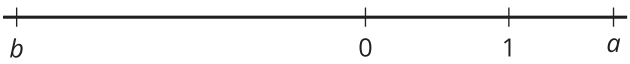
   b. Mai says the equation $3s = 42$ describes the situation. What does the variable s represent?

   c. Mai says that she can rewrite her equation as $s = \frac{1}{3} \cdot 42$ to solve for s. Compare Mai's strategy to your strategy for finding the number of students on last year's trip.

4. The cost of the hiking trip this year is $\frac{2}{3}$ of the cost of last year's trip. This year's trip cost $32. Write and solve an equation to find the cost of last year's trip.

Are you ready for more?

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other rational numbers a and b.



Decide which of the following numbers are positive and which are negative.

$a - 1$ $a - 2$ $-b$ $a + b$ $a - b$ $ab + 1$

20.4 Card Sort: Matching Inverses

Your teacher will give you a set of cards. Take turns with your partner to match a number with its additive inverse.

1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.
   Pause here for a class discussion.
3. Use the same cards and take turns with your partner to match a number with its multiplicative inverse.

Lesson 20 Summary

To solve the equation $x + 8 = -5$, we can add the opposite of 8, or -8, to each side:

$$\begin{aligned} x + 8 &= -5 \\ (x + 8) + -8 &= (-5) + -8 \\ x &= -13 \end{aligned}$$

Because adding the opposite of a number is the same as subtracting that number, we can also think of it as subtracting 8 from each side.

We can use the same approach for this equation:

$$\begin{aligned} -12 &= t + -\frac{2}{9} \\ (-12) + \frac{2}{9} &= \left(t + -\frac{2}{9}\right) + \frac{2}{9} \\ -11\frac{7}{9} &= t \end{aligned}$$

To solve the equation $8x = -5$, we can multiply each side by the reciprocal of 8, or $\frac{1}{8}$:

$$\begin{aligned} 8x &= -5 \\ \frac{1}{8}(8x) &= \frac{1}{8}(-5) \\ x &= -\frac{5}{8} \end{aligned}$$

Because multiplying by the reciprocal of a number is the same as dividing by that number, we can also think of it as dividing by 8.

We can use the same approach for this equation:

$$\begin{aligned} -12 &= -\frac{2}{9}t \\ -\frac{9}{2}(-12) &= -\frac{9}{2}\left(-\frac{2}{9}t\right) \\ 54 &= t \end{aligned}$$