Unit 4, Lesson 6

Equivalent Equations

Let's investigate what makes two equations equivalent.

6.1 Two Expressions

Your teacher will assign you one of these expressions:

$$\frac{n^2 - 9}{2(4 - 3)} \quad \text{or} \quad (n + 3) \cdot \frac{n - 3}{8 - 3 \cdot 2}$$

Evaluate your expression when n is:

1. 5

2. -1

3. With your partner, select 2 other values to try. Consider different kinds of numbers like fractions, decimals, large numbers, small numbers, negative numbers, and so on.

6.2 Much Ado about Ages

1. Write as many equations as possible that could represent the relationship between the ages of the two children in each family described. Be prepared to explain what each part of your equation represents.

   a. In Family A, the youngest child is 7 years younger than the oldest, who is 18.

   b. In Family B, the middle child is 5 years older than the youngest child.

2. Tyler thinks that the relationship between the ages of the children in Family B can be described with $2m - 2y = 10$, where m is the age of the middle child and y is the age of the youngest. Explain why Tyler is right.

3. Are any of these equations **equivalent** to one another? If so, which ones? Explain your reasoning.

   $3a + 6 = 15$ $3a = 9$ $a + 2 = 5$ $\frac{1}{3}a = 1$

Are you ready for more?

Here is a puzzle:

$$\begin{aligned} m + m &= N \\ N + N &= p \\ m + p &= Q \\ p + Q &= ? \end{aligned}$$

Which expressions could be equal to $p + Q$?

$2p + m$ $4m + N$ $3N$ $9m$

6.3 What's Acceptable?

Noah is buying a pair of jeans and using a coupon for 10% off. The total price is $56.70, which includes $2.70 in sales tax. Noah's purchase can be modeled by the equation:

$$x - 0.1x + 2.70 = 56.70$$

1. Discuss with a partner:
   a. What does the solution to the equation mean in this situation?
   b. How can you verify that 70 is not a solution but 60 is the solution?

2. Here are some equations that are related to $x - 0.1x + 2.70 = 56.70$. Each equation is a result of performing one or more moves on that original equation. Each can also be interpreted in terms of Noah's purchase.
   For each equation, determine either what move was made or how the equation could be interpreted. (Some examples are given here.) Then check if 60 is the solution of the equation.

   Equation A $100x - 10x + 270 = 5,670$

   - What was done?
   - Interpretation? The price is expressed in cents instead of dollars.
   - Same solution?

   Equation B $x - 0.1x = 54$

   - What was done? Subtract 2.70 from both sides of the equation.
   - Interpretation?
   - Same solution?

   Equation C $0.9x + 2.70 = 56.70$

   - What was done?
   - Interpretation? 10% off means paying 90% of the original price. 90% of the original price plus sales tax is $56.70.
   - Same solution?

3. Here are some other equations. For each equation, determine what move was made or how the equation could be interpreted. Then check if 60 is the solution to the equation.

   Equation D $x - 0.1x = 56.70$

   - What was done?
   - Interpretation? The price after using the coupon for 10% off and before sales tax is $56.70.
   - Same solution?

   Equation E $x - 0.1x = 59.40$

   - What was done? Subtract 2.70 from the left and add 2.70 to the right.
   - Interpretation?
   - Same solution?

   Equation F $2(x - 0.1x + 2.70) = 56.70$

   - What was done?
   - Interpretation? The price of 2 pairs of jeans, after using the coupon for 10% off and paying sales tax, is $56.70.
   - Same solution?

4. Which of the six equations are equivalent to the original equation? Be prepared to explain how you know.

Lesson 6 Summary

Suppose we bought 2 packs of markers and a $0.50 glue stick for $6.10. If p is the dollar cost of 1 pack of markers, the equation $2p + 0.50 = 6.10$ represents this purchase. The solution to this equation is 2.80.

Now suppose a friend bought 6 of the same packs of markers and 3 $0.50 glue sticks, and paid $18.30. The equation $6p + 1.50 = 18.30$ represents this purchase. The solution to this equation is also 2.80.

We can say that $2p + 0.50 = 6.10$ and $6p + 1.50 = 18.30$ are **equivalent equations** because they have exactly the same solution. Besides 2.80, no other values of p make either equation true. Only the price of \$2.80 per pack of markers satisfies the constraint in each purchase.

$2p + 0.50 = 6.10$	How do we write equivalent equations like these?
$6p + 1.50 = 18.30$	There are certain moves we can perform!

In this example, the second equation, $6p + 1.50 = 18.30$, is a result of multiplying each side of the first equation by 3. Buying 3 times as many markers and glue sticks means paying 3 times as much money. The unit price of the markers hasn't changed.

Here are some other equations that are equivalent to $2p + 0.50 = 6.10$, along with the moves that led to these equations.

- $2p + 4 = 9.60$ — Add 3.50 to each side of the original equation.
- $2p = 5.60$ — Subtract 0.50 from each side of the original equation.
- $\frac{1}{2}(2p + 0.50) = 3.05$ — Multiply each side of the original equation by $\frac{1}{2}$.
- $2(p + 0.25) = 6.10$ — Apply the distributive property to rewrite the left side.

In each case:

- The move is acceptable because it doesn't change the equality of the two sides of the equation. If $2p + 0.50$ has the same value as 6.10, then multiplying $2p + 0.50$ by $\frac{1}{2}$ and multiplying 6.10 by $\frac{1}{2}$ keep the two sides equal.
- Only $p = 2.80$ makes the equation true. Any value of p that makes an equation false also makes the other equivalent equations false. (Try it!)

These moves—applying the distributive property, adding the same amount to both sides, dividing each side by the same number, and so on—might be familiar because we have performed them when solving equations. Solving an equation essentially involves writing a series of equivalent equations that eventually isolates the variable on one side.

Not all moves that we make on an equation would create equivalent equations, however!

For example, if we subtract 0.50 from the left side but add 0.50 to the right side, the result is $2p = 6.60$. The solution to this equation is 3.30, not 2.80. This means that $2p = 6.60$ is *not* equivalent to $2p + 0.50 = 6.10$.