# Moving Functions

Let's represent vertical and horizontal translations using function notation.

## 2.1 What Happened to the Equation?

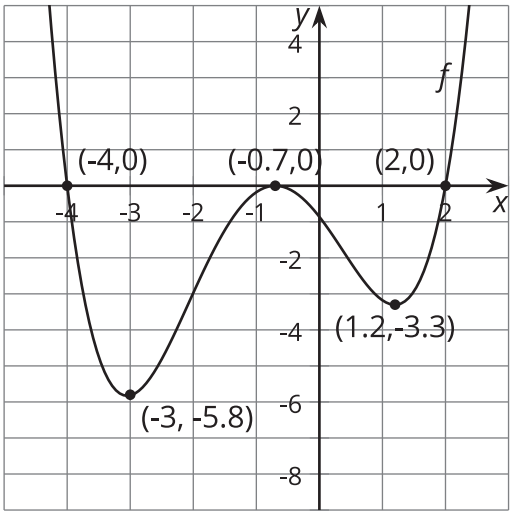
Graph each function using technology. Describe how to transform $f(x) = x^2(x-2)$ to get to the functions shown here, in terms of both the graph and the equation.

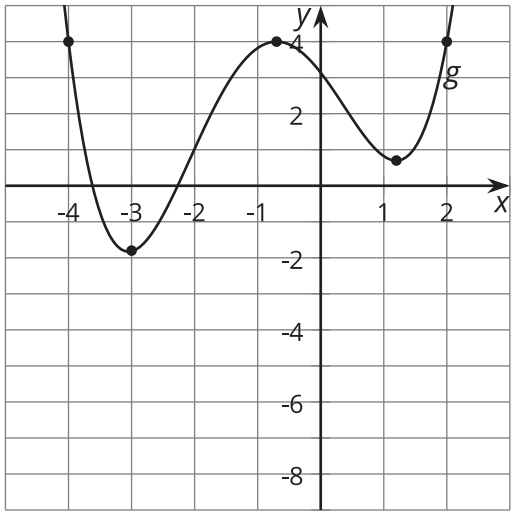
1. $h(x) = x^2(x-2) - 5$

2. $g(x) = (x-4)^2(x-6)$

# 2.2 Writing Equations for Vertical Translations

The graph of function $g$ is a vertical translation of the graph of polynomial $f$.




1. Complete the $g(x)$ column of the table.

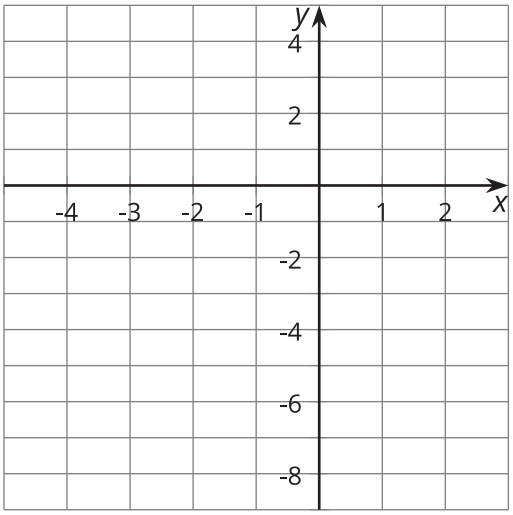
| $x$ | $f(x)$ | $g(x)$ | $h(x) = f(x) - 2.5$ |
|---|---|---|---|
| -4 | 0 | | |
| -3 | -5.8 | | |
| -0.7 | 0 | | |
| 1.2 | -3.3 | | |
| 2 | 0 | | |

2. If $f(0) = -0.86$, what is $g(0)$? Explain how you know.

3. Write an equation for $g(x)$ in terms of $f(x)$ for any input $x$.

4. The function $h$ can be written in terms of $f$ as $h(x) = f(x) - 2.5$. Complete the $h(x)$ column of the table.

5. Sketch the graph of function $h$.

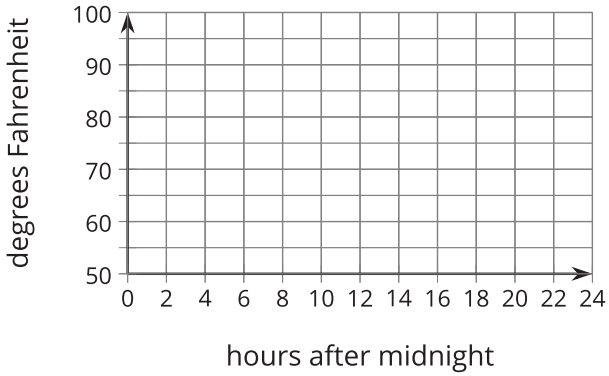



6. Write an equation for $g(x)$ in terms of $h(x)$ for any input $x$.

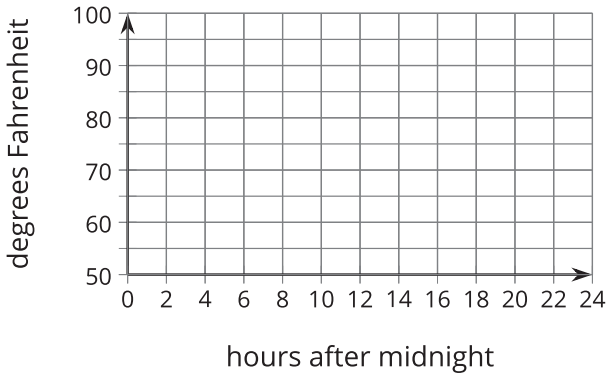
# 2.3 Heating the Kitchen

A bakery kitchen has a thermostat set to $65^\circ\text{F}$. Starting at 5:00 a.m., the temperature in the kitchen rises to $85^\circ\text{F}$ when the ovens and other kitchen equipment are turned on to bake the daily breads and pastries. The ovens are turned off at 10:00 a.m. when the baking finishes.

1. Sketch a graph of the function $H$ that gives the temperature in the kitchen $H(x)$, in degrees Fahrenheit, $x$ hours after midnight.




2. The bakery owner decides to change the shop hours to start and end 2 hours earlier. This means the daily baking schedule will also start and end two hours earlier. Sketch a graph of the new function $G$, which gives the temperature in the kitchen as a function of time.




3. Explain what $H(10.25) = 80$ means in this situation. Why is this reasonable?

4. If $H(10.25) = 80$, then what would the corresponding point on the graph of $G$ be? Use function notation to describe the point on the graph of $G$.

5. Write an equation for $G$ in terms of $H$. Explain why your equation makes sense.

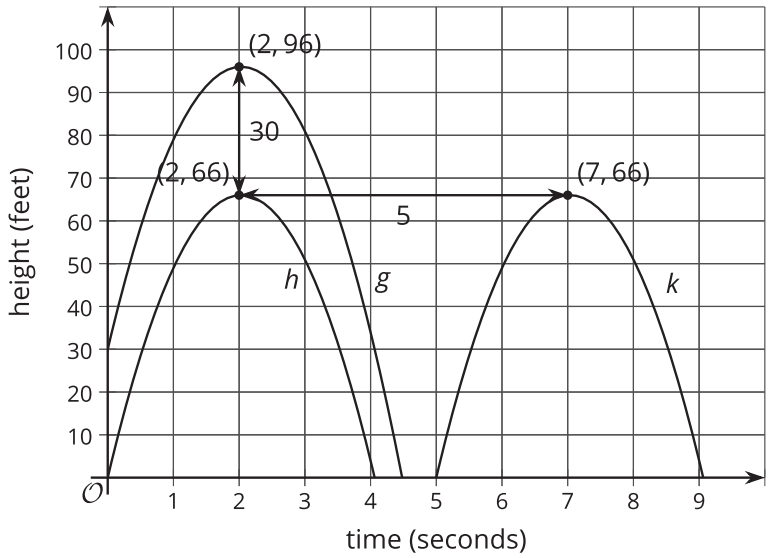
## Are you ready for more?

Write an equation that defines your piecewise function, $H$, algebraically.

## Lesson 2 Summary

A pumpkin catapult is used to launch a pumpkin vertically into the air. The function $h$ gives the height $h(t)$, in feet, of this pumpkin above the ground $t$ seconds after launch.

Now consider what happens if the pumpkin had been launched at the same time, but from a platform 30 feet above the ground. Let function $g$ represent the height $g(t)$, in feet, of this pumpkin. How would the graphs of $h$ and $g$ compare?




Since the height of the second pumpkin is 30 feet greater than the first pumpkin at all times $t$, the graph of function $g$ is translated up 30 feet from the graph of function $h$. For example, the point $(2, 66)$ on the graph of $h$ tells us that $h(2) = 66$, so the original pumpkin was 66 feet high after 2 seconds. The new pumpkin would be 30 feet higher than that, so $g(2) = 96$. Since all the outputs of $g$ are 30 more than the corresponding outputs of $h$, we can express $g(t)$ in terms of $h(t)$, using function notation as $g(t) = h(t) + 30$.

Now suppose instead the pumpkin launched 5 seconds later. Let function $k$ represent the height $k(t)$, in feet of this pumpkin. The graph of $k$ is translated right 5 seconds from the graph of $h$. We can also say that the output values of $k$ are the same as the output values of $h$ 5 seconds earlier. For example, $k(7) = 66$ and $h(7 - 5) = h(2) = 66$. This means we can express $k(t)$ in terms of $h(t)$ as $k(t) = h(t - 5)$.

