

What Is a Logarithm?

Let's learn about logarithms.

9.1 Math Talk: Finding Solutions

Find or estimate the value of each variable mentally.

- $4^a = 16$

- $4^b = 2$

- $4^{\frac{5}{2}} = c$

- $4^d = 56$

9.2

A Table of Numbers

x	$\log_{10}(x)$
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1

x	$\log_{10}(x)$
20	1.3010
30	1.4771
40	1.6021
50	1.6990
60	1.7782
70	1.8451
80	1.9031
90	1.9542
100	2

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3

x	$\log_{10}(x)$
2,000	3.3010
3,000	3.4771
4,000	3.6021
5,000	3.6990
6,000	3.7782
7,000	3.8451
8,000	3.9031
9,000	3.9542
10,000	4

- Analyze the table and discuss with a partner what you think the table tells us.
- Use the table to find the value of the unknown exponent that makes each equation true.
 - $10^w = 1,000$
 - $10^y = 9$
 - $10^z = 90$
- Notice that some values in the columns labeled $\log_{10} x$ are whole numbers, but most are decimals. Why do you think that is?



9.3 Hello, Logarithm!

1. Here are two true equations based on the information from the table:

$$\log_{10} 100 = 2$$

$$\log_{10} 1,000 = 3$$

What values could replace the “?” in these equations to make them true?

- $\log_{10} 1,000,000 = ?$
 - $\log_{10} 1 = ?$
 - $\log_{10} ? = 7$
2. Estimate the value of $\log_{10} 610$? Be prepared to explain how you know.
3. The term *log* is short for **logarithm**. Discuss the following questions with a partner, and record your responses:
- What do you think logarithm means or does?
 - Next to “log” is a subscript—a number or letter printed smaller and below the line of text—that is called the “base.” What do you think the base of the logarithm tells us?
 - What about the other two numbers on either side of the equal sign (for example, the 100 and the 2 in $\log_{10} 100 = 2$)? What do they tell us?

Are you ready for more?

- Let x be a number. For which x is $\log_{10}(x)$ an integer?
- Use the table to write an approximate value for $\log_{10}(900,000)$. Explain your reasoning.

Lesson 9 Summary

We know how to solve equations such as $10^a = 10,000$ or $10^b = \frac{1}{100}$ by thinking about integer powers of 10. The solutions are $a = 4$ and $b = -2$. What about an equation such as $10^p = 250$?

Because $10^2 = 100$ and $10^3 = 1,000$, we know that p is between 2 and 3. We can use a **logarithm** to represent the exact solution to this equation and write it as:

$$p = \log_{10} 250$$

The expression $\log_{10} 250$ is read “the log, base 10, of 250.”

- The small, slightly lowered “10” refers to the base of 10.
- The 250 is the value of the power of 10.
- $\log_{10} 250$ is the value of the exponent p that makes 10^p equal 250.

In the specific case where the base of the logarithm is 10, the “log” can be written without the number 10. For example, $\log_{10} 250$ can also be written as $\log 250$, and this expression is read “the log of 250.”

One way to estimate logarithms is with a logarithm table. For example, using this base 10 logarithm table we can see that $\log_{10} 250$ is between 2.3010 and 2.4771.

x	$\log_{10}(x)$
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