Unit 4, Lesson 18

Logarithm Change of Base Rule

Let's approximate logarithms in other bases.

18.1 Log as a Power

Use your understanding of logarithms to find the value of each expression. Be prepared to explain your reasoning.

1. $2^{\log_2(8)}$
2. $5^{\log_5(25)}$
3. $3^{\log_3(81)}$
4. $2^{\log_2(10)}$
5. $10^{\log(65)}$
6. $e^{\ln(4)}$

18.2 Changing the Logarithm Base

The **change of base rule** for logarithms states that, for any positive number c (except 1),

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

1. Use the change of base rule to rewrite each of these logarithms so that $c = 10$.

   a. $\log_3(9)$

   b. $\log_2(6)$

   c. $\log_5(20)$

   d. $\log_6(1{,}000)$

2. Use your calculator to approximate the values of the logarithms in the previous question. Then check your approximation using exponents.

3. If the rule should work for any base, then it should also work to change the logarithms into base e. Select one of the logarithms to change into a logarithm with base e, then use your calculator to check that it has the same approximation.

Are you ready for more?

Why can't the base of a logarithm be 1?

1. Rewrite this equation using exponents: $\log_1(4) = x$. What value of x could make that equation true?

2. Rewrite this equation using exponents: $\log_1(1) = y$. What value of y could make that equation true?

18.3 Proving the Change of Base Rule

Here are some steps for a proof of the change of base rule: $\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$.

1. Start with the expression $\log_c\left(a^{\log_a(b)}\right)$. Rewrite this expression in two ways.
   a. Use the power rule for logarithms.
   b. Rewrite the expression $a^{\log_a(b)}$ so that it does not use a power.
2. Set the two expressions you wrote equal to each other. How can you turn this equation into the change of base rule?

Lesson 18 Summary

The **change of base rule** allows us to rewrite any logarithm to have any other positive value (except 1) for the base. We can use this equation to change the base: $\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$.

Many calculators can calculate logarithms only in base 10 ($\log$) or base e ($\ln$). The change of base rule can be used to change any logarithm into one of these bases so we can find approximate values of any logarithm.

For example, we know that $\log_2(5)$ is between 2 and 3 because $2^2 = 4$ and $2^3 = 8$, but it requires a lot of guessing and checking to get closer to an actual value when our calculator can't compute logarithms in base 2. Using the change of base rule allows us to rewrite the logarithm as $\log_2(5) = \frac{\log(5)}{\log(2)}$, and this can be entered into most scientific calculators to get about 2.3219. We could have also changed into logarithms with base e to get $\frac{\ln(5)}{\ln(2)}$, which has the same approximate value.