Odd and Even Numbers

Let's explore even and odd numbers.

21.1 Math Talk: Evens and Odds

Evaluate mentally.

- $64 + 88$
- $65 + 89$
- $14 \cdot 5$
- $14 \cdot 4$

21.2 Always Even, Never Odd

Here are some statements about the sums and products of numbers. For each statement:

- Decide whether it is *always* true, true for *some* numbers but not others, or *never* true.
- Use examples to explain your reasoning.

1. Sums:
    a. The sum of 2 even numbers is even.

    b. The sum of an even number and an odd number is odd.

    c. The sum of 2 odd numbers is odd.

2. Products:
    a. The product of 2 even numbers is even.

    b. The product of an even number and an odd number is odd.

    c. The product of 2 odd numbers is odd.

21.3 Even + Odd = Odd

How do we know that the sum of an even number and an odd number *must* be odd? Examine this proof and answer the questions throughout.

Let a represent an even number, b represent an odd number, and s represent the sum $a+b$.

1. What does it mean for a number to be even? Odd?

   Assume that s is even, then we will look for a reason the original statement cannot be true. Since a and s are even, we can write them as 2 times an integer. Let $a=2k$ and $s=2m$ for some integers k and m.

2. Can this always be done? To convince yourself, write 4 different even numbers. What is the value for k for each of your numbers when you set them equal to $2k$?

   Then we know that $a+b=s$ and $2k+b=2m$.

   Divide each side by 2 to get that $k+\frac{b}{2}=m$.

   Rewrite the equation to get $\frac{b}{2}=m-k$.

   Since m and k are integers, then $\frac{b}{2}$ must be an integer as well because the difference of 2 integers is an integer.

3. Is the difference of 2 integers always an integer? Select 4 pairs of integers and subtract them to convince yourself that their difference is always an integer.

4. What does the equation $\frac{b}{2}=m-k$ tell us about $\frac{b}{2}$? What does that mean about b?

5. Look back at the original description of b. What is wrong with what we have found?

The logic for everything in the proof works, so the only thing that could've gone wrong was our assumption that s is even. Therefore, s must be odd.

