Lesson 13: Completing the Square (Part 2)

- Let’s solve some harder quadratic equations.

13.1: Math Talk: Equations with Fractions

Solve each equation mentally.

$x + x = \frac{1}{4}$

$(\frac{3}{2})^2 = x$

$\frac{3}{5} + x = \frac{9}{5}$

$\frac{1}{12} + x = \frac{1}{4}$

13.2: Solving Some Harder Equations

Solve these equations by completing the square.

1. $(x - 3)(x + 1) = 5$

2. $x^2 + \frac{1}{2}x = \frac{3}{16}$

3. $x^2 + 3x + \frac{8}{4} = 0$

4. $(7 - x)(3 - x) + 3 = 0$

5. $x^2 + 1.6x + 0.63 = 0$

Are you ready for more?

1. Show that the equation $x^2 + 10x + 9 = 0$ is equivalent to $(x + 3)^2 + 4x = 0$.

2. Write an equation that is equivalent to $x^2 + 9x + 16 = 0$ and that includes $(x + 4)^2$.

3. Does this method help you find solutions to the equations? Explain your reasoning.

13.3: Spot Those Errors!

Here are four equations, followed by worked solutions of the equations. Each solution has at least one error.

- Solve one or more of these equations by completing the square.
- Then, look at the worked solution of the same equation as the one you solved. Find and describe the error or errors in the worked solution.

1. $x^2 + 14x = -24$

2. $x^2 - 10x + 16 = 0$

3. $x^2 + 2.4x = -0.8$

4. $x^2 - \frac{6}{5}x + \frac{1}{5} = 0$

Worked solutions (with errors):

1.

$$\begin{aligned} x^2 + 14x &= -24 \\ x^2 + 14x + 28 &= 4 \\ (x+7)^2 &= 4 \end{aligned}$$

$$\begin{aligned} x + 7 &= 2 \quad \text{or} \quad x + 7 = -2 \\ x &= -5 \quad \text{or} \quad x = -9 \end{aligned}$$

2.

$$\begin{aligned} x^2 - 10x + 16 &= 0 \\ x^2 - 10x + 25 &= 9 \\ (x-5)^2 &= 9 \end{aligned}$$

$$\begin{aligned} x - 5 &= 9 \quad \text{or} \quad x - 5 = -9 \\ x &= 14 \quad \text{or} \quad x = -4 \end{aligned}$$

3.

$$\begin{aligned} x^2 + 2.4x &= -0.8 \\ x^2 + 2.4x + 1.44 &= 0.64 \\ (x+1.2)^2 &= 0.64 \\ x + 1.2 &= 0.8 \\ x &= -0.4 \end{aligned}$$

4.

$$\begin{aligned} x^2 - \frac{6}{5}x + \frac{1}{5} &= 0 \\ x^2 - \frac{6}{5}x + \frac{9}{25} &= \frac{9}{25} \\ \left(x - \frac{3}{5}\right)^2 &= \frac{9}{25} \end{aligned}$$

$$\begin{aligned} x - \frac{3}{5} &= \frac{3}{5} \quad \text{or} \quad x - \frac{3}{5} = -\frac{3}{5} \\ x &= \frac{6}{5} \quad \text{or} \quad x = 0 \end{aligned}$$

Lesson 13 Summary

Completing the square can be a useful method for solving quadratic equations in cases in which it is not easy to rewrite an expression in factored form. For example, let’s solve this equation:

$$x^2 + 5x - \frac{75}{4} = 0$$

First, we’ll add $\frac{75}{4}$ to each side to make things easier on ourselves.

$$\begin{aligned} x^2 + 5x - \frac{75}{4} + \frac{75}{4} &= 0 + \frac{75}{4} \\ x^2 + 5x &= \frac{75}{4} \end{aligned}$$

To complete the square, take $\frac{1}{2}$ of the coefficient of the linear term 5, which is $\frac{5}{2}$, and square it, which is $\frac{25}{4}$. Add this to each side:

$$\begin{aligned} x^2 + 5x + \frac{25}{4} &= \frac{75}{4} + \frac{25}{4} \\ x^2 + 5x + \frac{25}{4} &= \frac{100}{4} \end{aligned}$$

Notice that $\frac{100}{4}$ is equal to 25 and rewrite it:

$$x^2 + 5x + \frac{25}{4} = 25$$

Since the left side is now a perfect square, let's rewrite it:

$$\left(x + \frac{5}{2}\right)^2 = 25$$

For this equation to be true, one of these equations must true:

$$x + \frac{5}{2} = 5 \quad \text{or} \quad x + \frac{5}{2} = \text{-}5$$

To finish up, we can subtract $\frac{5}{2}$ from each side of the equal sign in each equation.

$$\begin{aligned} x = 5 - \frac{5}{2} &\quad \text{or} \quad x = \text{-}5 - \frac{5}{2} \\ x = \frac{5}{2} &\quad \text{or} \quad x = \text{-}\frac{15}{2} \\ x = 2\frac{1}{2} &\quad \text{or} \quad x = \text{-}7\frac{1}{2} \end{aligned}$$

It takes some practice to become proficient at completing the square, but it makes it possible to solve many more equations than you could by methods you learned previously.