



Connecting Similarity and Transformations

Goals

- Comprehend the definition of similarity in terms of dilations and rigid transformations.
- Generate similarity statements about similar figures, and identify proportional relationships among corresponding side lengths (in writing).

Learning Targets

- I can write similarity statements.
- I know the definition of similarity.

Lesson Narrative

In this lesson, students develop an understanding of how **similar** figures are pairs of figures for which there is a sequence of rigid transformations and a dilation that takes one figure onto the other. Then students practice writing a sequence of transformations that could be used to take one triangle onto a similar triangle and identify the scale factor used for the dilation. They must use the structure of transformations and dilation to write out the sequence of transformations (MP7).

Standards

Building On	HSG-CO.B.6
Addressing	HSG-SRT.A.1.b, HSG-SRT.A.2, HSG-SRT.B.5
Building Toward	HSG-SRT.A.2

Instructional Routines

- Card Sort
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display

Required Materials

Materials to Gather

- Rulers: Activity 1, Activity 2
- Geometry toolkits (HS): Activity 2

Materials to Copy

- Not-So-Rigid Transformations Cards (1 copy for every 2 students): Activity 2

Required Preparation

Activity 3:

Students will continue adding to their reference chart in this activity. Be prepared to add to the class display. The Blank Reference Chart for students and a teacher copy of a completed version are available in the blackline masters for the unit.

If there are multiple sections of this course in the same classroom, consider hiding entries on the class reference chart and revealing them at the appropriate time rather than making multiple displays.



Student Facing Learning Goals

 Let's identify similar figures.

6.1

Dilation Miscalculation

Warm-up

 5 min

Activity Narrative

In a subsequent lesson, students will justify why not all rectangles are similar to each other and will explore a false proof that all rectangles are similar. This activity previews that thinking, as students explain what went wrong with this attempt to dilate a square. Monitor for students who:

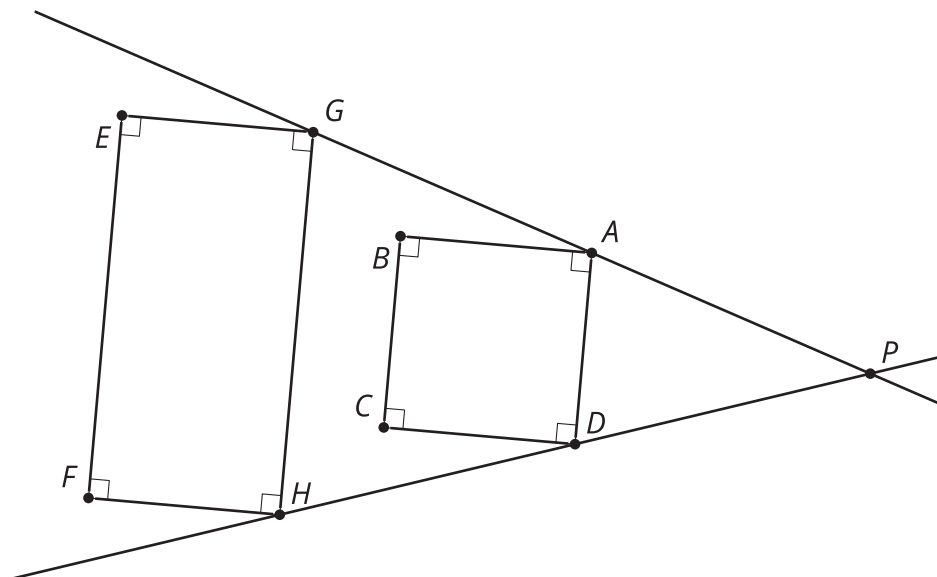
- Notice that the figures are not the same shape.
- Notice that the sides are not scaled by the same scale factor.
- Notice that E and F are not on rays PB and PC .

Standards

Addressing HSG-SRT.A.1.b

Building Toward HSG-SRT.A.2

Student Task Statement



How do you know that $GHFE$ is not a dilation of $ADCB$?

Student Response

Sample response: The angles are right, but the side lengths are not in the same proportion. They need to use the same scale factor for all four sides.

Activity Synthesis

The goal of this *Activity Synthesis* is for students to begin to justify why accurate dilations lead to figures with all pairs of corresponding angles congruent and all pairs of corresponding sides having the same ratio as the scale factor. They approach this from the contrapositive: If a figure does not have all pairs of corresponding angles congruent and all pairs of corresponding sides having the same ratio as the scale factor, then the figures cannot be a dilation of one another.

Invite students to share who:

- Noticed that the figures are not the same shape.
- Noticed that the sides are not scaled by the same scale factor.
- Noticed that E and F are not on rays PB and PC .

If scale factor or angles are not mentioned by students, ask, “All the angles are the same, doesn’t that mean they’re similar?” (No. Angles need to be congruent *and* sides need to be in the same ratio.) If no student noticed that E and F are not on rays PB and PC , draw in the auxiliary lines. The goal is for students to specify the properties that must be true if a dilation is to exist between two figures.

6.2

Card Sort: Not-So-Rigid Transformations

15 min

Activity Narrative

Students sort different pairs of figures during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and to make connections (MP2, MP7).

Monitor for different ways groups choose to categorize the pairs of figures, but especially for categories that distinguish between congruent figures, similar figures, and neither congruent nor similar.

After doing and discussing a card sort, students are assigned one pair of figures to figure out a sequence of rigid motions and dilations that takes one figure onto the other. Students use their prior knowledge of rigid transformations and their experience carrying out dilations to help them be successful in this task.



Standards

Addressing HSG-SRT.A.2



Instructional Routines

- Card Sort
- MLR2: Collect and Display

Launch

Arrange students in groups of 2.

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2, and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

- Give students 1 minute to place all the cards face up and to start thinking about possible ways to sort the cards into categories.
- Pause the class, and select 1–3 students to share the categories they identified.
- Discuss as many different categories as time allows.



Attend to the language that students use to describe their categories and pairs of figures, giving them opportunities to describe their pairs of figures more precisely. Highlight the use of terms like “quadrilaterals,” “congruent,” and “similar.” After a brief discussion, invite students to open their books or devices and continue with the activity.

After students have sorted their cards, assign each group a card to write transformations. Note: Do not assign the rhombi to any groups.



Access for English Language Learners

MLR2 Collect and Display. Collect the language that students use to sort the cards. Display words and phrases, such as “similar,” “dilation,” “copy,” and “bigger version.” During the *Activity Synthesis*, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display, as needed.

Advances: Conversing, Reading



Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as

- “First, I ____ because ____.”
- “I noticed ____, so I ____.”
- “Why did you ____?”

Supports accessibility for: Language, Social-Emotional Functioning



Student Task Statement

1. Your teacher will give you a set of cards. Sort the cards into categories of your choosing. Be prepared to explain the meaning of your categories.
2. Your teacher will assign you one card. Write the sequence of transformations (translation, rotation, reflection, dilation) to take one figure to the other.
3. What scale factor is used to go from Figure F to Figure G ? What scale factor is used to go from Figure G to Figure F ?

Student Response

1. Sample responses: triangles versus quadrilaterals, congruent versus not congruent, dilations versus not dilations
2. Sample responses:
 - Card 1: Dilate triangle ROQ using O as the center and a scale factor of $\frac{3}{2}$.
 - Card 2: Dilate Figure G using the common vertex as the center and a scale factor of 4. Reflect the image across the shared side.
 - (Card 3 should not have been assigned)
 - Card 4: Label a vertex on Figure F as A and its corresponding vertex on Figure G as B . Rotate Figure F 90 degrees clockwise using center A . Translate the image by the directed line segment AB .
 - Card 5: Label a vertex on Figure F as A and its corresponding vertex on Figure G as B . Dilate Figure F using center A and a scale factor of $\frac{3}{4}$. Translate the image by the directed line segment AB . Rotate the image using center B until the triangles coincide.



- Card 6: Label the right angle on Figure F as R and the right angle on Figure G as P . Label another vertex on Figure F as A and its corresponding vertex on Figure G as B . Translate Figure F by the directed line segment RP . Rotate the image using center P until A' and B coincide. Reflect the resulting image across line PB .

3. Card 1: $\frac{2}{3}$ and $\frac{3}{2}$, Card 2: $\frac{1}{4}$ and 4, Card 4: 1 and 1, Card 5: $\frac{3}{4}$ and $\frac{4}{3}$, Card 6: 1 and 1

Building on Student Thinking

If students struggle to describe their transformations, remind them that they can use tracing paper to redraw the figures and add labels to the points.

If students struggle to name the scale factor for congruent figures, ask them to describe what they know about scale factors that make the image larger (the scale factors are greater than 1), then describe scale factors that make the image smaller (the scale factors are less than 1). Then, ask what scale factor might work if the image is the same size.



Are You Ready for More?



For your card, find a sequence of transformations that takes Figure G to Figure F . How does this sequence compare to the sequence that took Figure F to Figure G ?

Extension Student Response

Sample response: The new sequence is the reverse of the last. So I can do all the same steps, but backward. Also, before I needed a scale factor that was less than one, but this time, it's greater than one.

Activity Synthesis

The purpose of this discussion is for students to describe sequences of rigid motions and dilations that show that two figures are either congruent or similar.

Record the scale factors for dilating from Figure F to Figure G , and from Figure G to Figure F , for each pair of figures except the rhombi. Include the right triangles and the trapezoids, which have a scale factor of 1. Ask students what they notice. (There's a scale factor either way. The scale factors are multiplicative inverses. The congruent figures have a scale factor of 1.)

6.3

Alphabet Soup

🕒 15 min

Activity Narrative

The goal of this activity is for students to use the definition of similarity, and to practice applying to similar figures some of the skills they learned when studying congruent figures. Students practice writing a sequence of rigid motions and dilations to take one figure onto another, write accurate similarity statements about the figures, and reason about what must be true about corresponding side lengths.



Standards

Building On HSG-CO.B.6



Instructional Routines

- MLR1: Stronger and Clearer Each Time



Launch

Add the following definition to the class reference chart, and ask students to add it to their reference charts:

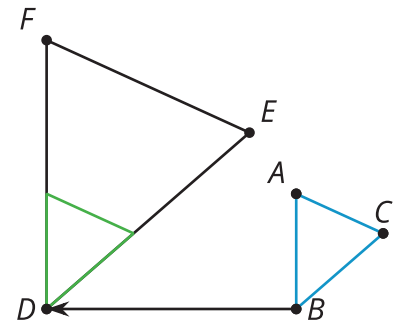
One figure is **similar** to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second.

(Definition)

Translation and dilation takes

$\triangle ABC$ onto $\triangle FDE$, so

$\triangle ABC \sim \triangle FDE$



Tell students that $\triangle ABC \sim \triangle FDE$ is a similarity statement, and we read $\sim$ as "is similar to."

Arrange students in groups of 2. After quiet work time, ask students to compare their responses to their partner's and to decide if they are both correct, even if they are different. Follow with a whole-class discussion.



Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to finding a sequence of transformations to show similarity. Invite listeners to ask questions and give feedback that will help clarify and strengthen their partner's ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening



Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge. To help students recall the symbols used in the image caption for parallel and perpendicular, ask "How else does the image show that the two segments are perpendicular?" (with a right angle)

Supports accessibility for: Memory, Language

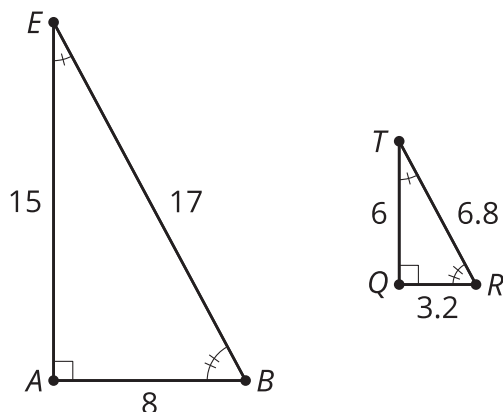


Student Task Statement



Are the triangles **similar**?

$$\overline{AB} \parallel \overline{QR}, \overline{AB} \perp \overline{AE}, \overline{QR} \perp \overline{QT}, \angle B \cong \angle R, \angle E \cong \angle T$$



1. Write a sequence of transformations (dilation, translation, rotation, reflection) to take one triangle to the other.
2. Write a similarity statement about the 2 figures, and explain how you know they are similar.
3. Compare your statement with your partner's statement. Is there more than one correct way to write a similarity statement? Is there a wrong way to write a similarity statement?

Student Response

1. Sample response: Translate figure ABE by the directed line segment from A to Q . Dilate figure $A'B'E'$ using center A' (or Q) and scale factor 0.4.
2. Sample response: Triangle ABE is similar to triangle QRT or $\triangle ABE \sim \triangle QRT$. The figures are similar because a sequence of rigid motions and dilations takes one onto the other.
3. Sample response: There are many correct ways to write similarity statements as long as the corresponding parts match. For example, these two triangles could be written: $BEA \sim RTQ$, $AEB \sim QTR$, or $EBA \sim TRQ$. It is wrong to write a similarity statement with the corresponding parts not matching. For example, $ABE \sim RTQ$ is incorrect.

Building on Student Thinking

If students struggle to define sequences of rigid motions and dilations, review an example from the card sort.

Activity Synthesis

Ask students to determine if their sequence of rigid motions and dilations will always work for any pair of triangles that have all pairs of corresponding angles congruent and all pairs of corresponding side lengths in the same proportion. (Probably not, some figures would require rotations and dilations. But the general method of dilating and using rigid motions will always work.)

For students who dilated first, ask:

- "How do you choose the dilation to use?" (Dilate from any vertex, by using the scale factor given by the ratio of a pair of corresponding sides.)
- "Once you dilate, how do you know you'll be able to use rigid motions to take one triangle to the other?" (After dilation, the triangles are congruent because their corresponding angles remain congruent, and all the



corresponding side lengths will be the same. Because they are congruent, there is always a sequence of rigid motions that takes one figure to the other. In this example, the lines are parallel and the triangles are oriented the same way, so only a translation is needed to align the figures.)

For students who used rigid motions first, ask:

- “How do you know rigid motion will line up parts of the figures?” (Because corresponding angles are congruent and one pair of corresponding sides is parallel, we’ll be able to line up both of the rays that meet at the vertex with a translation, because you can always take an angle onto another congruent angle.)
- “How do you know that dilation will then line up the remaining two points?” (Dilations keep angles the same measure, keep the center of the dilation in the same place, and move other points along the ray connecting the center to the point. Because the corresponding sides are all in the same proportion, if the right scale factor is used, then all of the vertex points should be in the same position so that the triangles line up.)

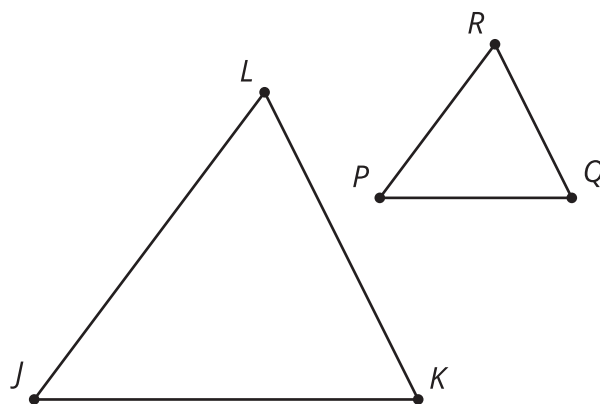
Lesson Synthesis

The goal of this discussion is to make sure that students are comfortable asserting that if all pairs of corresponding angles are congruent and all pairs of corresponding sides have lengths in the same proportion, that they could definitely find a sequence of rigid motions and a dilation that takes one triangle onto the other. Students will write more formal proofs of this idea in a subsequent lesson.

Invite students to choose a method of transforming one similar figure onto the other that they prefer, by either dilating first or using rigid motions first. Arrange students with a partner who prefers the same method.

Display a pair of triangles where one has sides twice as long as the other, and all the pairs of corresponding angles are congruent. Give students 1 minute to talk with their partner about their strategy for lining up the triangles.

$$JL = 2 \cdot PR$$



Ask students to find a group that preferred a different strategy than theirs. Invite each group to describe to the other how they will line up the triangles and how they know their strategy will work. (Dilate the smaller triangle by a scale factor of 2. The triangles will be congruent. Then you can translate, rotate, and reflect so that the triangles line up. Alternatively, use rigid motions to line up one pair of corresponding vertices and two pairs of corresponding sides. Then dilate the smaller triangle by a scale factor of 2.)

Invite students to explain why they are convinced that if two triangles have all pairs of corresponding angles congruent and all pairs of corresponding sides similar, the triangles must be similar.



6.4

Forward and Backward?

Cool-down

5 min

Standards

Addressing HSG-SRT.A.2, HSG-SRT.B.5

Student Task Statement

Determine if each statement must be true, could possibly be true, or definitely can't be true. Explain or show your reasoning.

1. Congruent figures are similar.
2. Similar figures are congruent.

Student Response

Sample responses:

1. Must be true. Any congruent figures are similar because they can be taken onto each other using transformations.
2. Could possibly be true. Some similar figures are congruent, but only if the dilation uses a scale factor of 1.

Responding to Student Thinking

Points to Emphasize

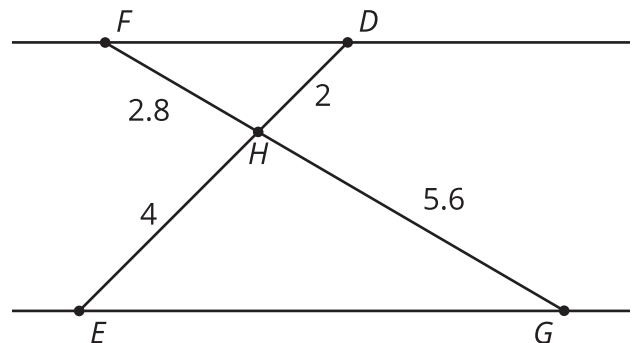
If most students struggle with the idea that congruent figures are similar figures that have a scale factor of 1, focus on this concept as opportunities arise in upcoming lessons. For example, after students have proven that all circles are similar, ask them whether congruent circles are also similar, and ask them to explain why.

Geometry, Unit 3, Lesson 8, Activity 3 Always? Prove It!

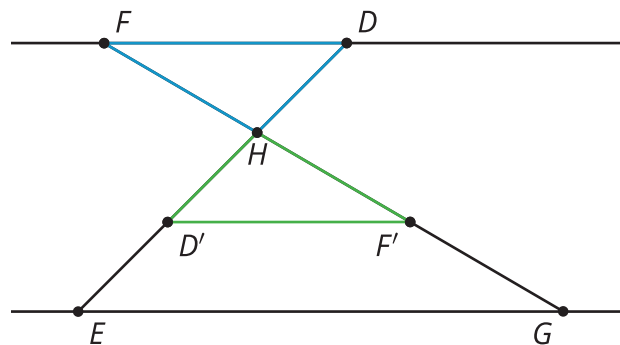
Lesson 6 Summary

One figure is **similar** to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second. For example, triangle DHF is similar to triangle EHG ($\triangle DHF \sim \triangle EHG$).

What is a rotation and a dilation that will take DHF onto EHG ?



The triangles are similar because a 180° rotation of DHF using center H will take segment HF onto segment HG , because 180° rotations take lines through the center of the rotation to themselves. It will also take segment HD onto segment HE for the same reason. Then G will be on a ray from H through F' , and E will be on a ray from H through D' . Because $\frac{H'F'}{HG} = \frac{H'D'}{HE} = \frac{1}{2}$, a dilation by a scale factor of 2 and center H will take $D'H'F'$ to fit exactly onto EHG . Therefore, there is a sequence of rigid motions and dilations that takes DHF onto EHG , and $\triangle DHF \sim \triangle EHG$.



- Because both rigid motions and dilations leave corresponding angles congruent, all pairs of corresponding angles in similar figures are congruent. In this example, angle D is congruent to angle E , angle F is congruent to angle G , and angle DHF is congruent to angle EHG .
- Because rigid motions keep lengths congruent and dilations scale them at the same proportion, all pairs of corresponding sides in similar figures are in the same proportion. In this example, $\frac{HE}{HD} = \frac{HG}{HF} = \frac{EG}{DF}$.

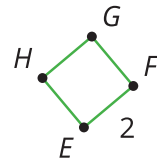
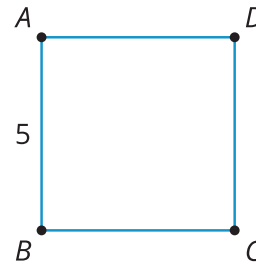
Glossary

• similar

Lesson 6 Practice Problems

1 Student Task Statement

Find a sequence of rigid motions and dilations that takes square $ABCD$ to square $EFGH$.



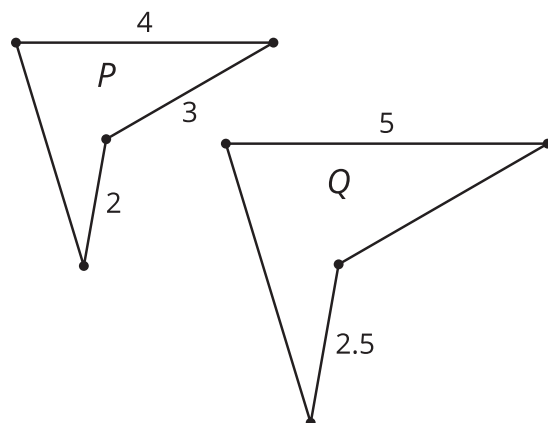
Solution

Sample response: Translate $ABCD$ by the directed segment AE , which will take B to a point B' . Then rotate the image counterclockwise using center E by angle $B'EF$. Finally, dilate using center E and a scale factor of $\frac{2}{5}$.

2 Student Task Statement

Quadrilaterals Q and P are similar.

- What is the scale factor of the dilation that takes P to Q ?
- What is the scale factor of the dilation that takes Q to P ?



Solution

- $\frac{5}{4}$
- $\frac{4}{5}$

3 Student Task Statement

What is our definition of "similarity"?



- A. If two figures have the same angles, then they are similar.
- B. If two figures have proportional side lengths, then they are similar.
- C. If there is a sequence of rigid transformations taking one figure to another, then they are similar.
- D. If there is a sequence of rigid transformations and dilations that take one figure to the other, then they are similar.

Solution

D

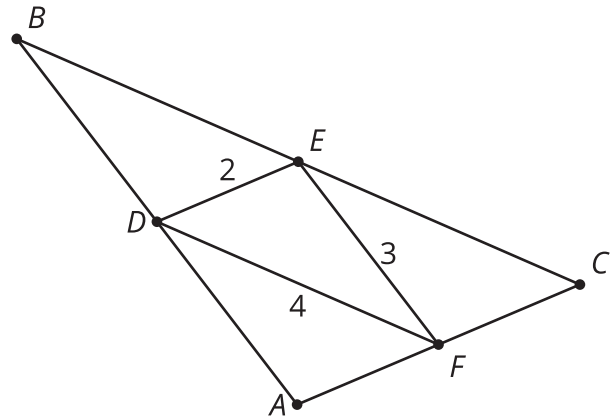
4

from Unit 3, Lesson 5



Student Task Statement

Triangle DEF is formed by connecting the midpoints of the sides of triangle ABC . The lengths of the sides of DEF are shown. What is the length of BC ?



- A. 3 units
- B. 4 units
- C. 6 units
- D. 8 units

Solution

D

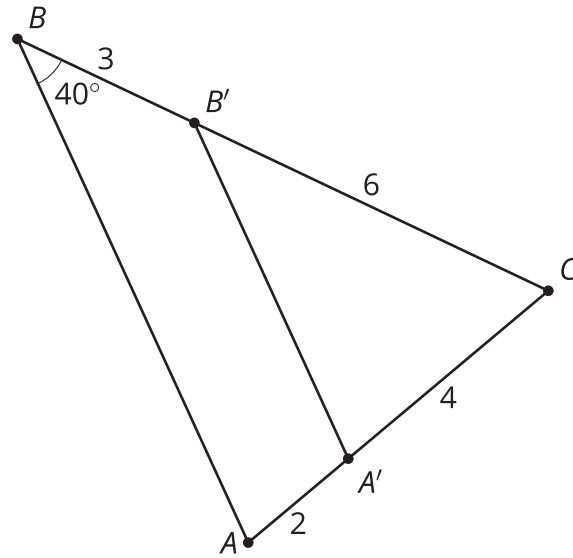
5

from Unit 3, Lesson 5



Student Task Statement

If AB is 12, what is the length of $A'B'$?



Solution

8

6

from Unit 3, Lesson 4



Student Task Statement

Right angle ABC is taken by a dilation with center P and a scale factor of $\frac{1}{2}$ to angle $A'B'C'$. What is the measure of angle $A'B'C'$?

Solution

 90°

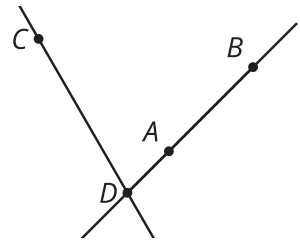
7

from Unit 3, Lesson 4

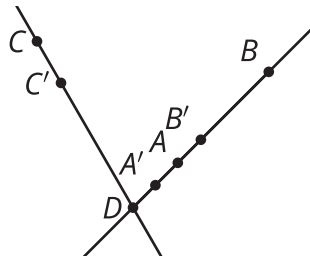


Student Task Statement

- a. Dilate point C using center D and scale factor $\frac{3}{4}$.
- b. Dilate segment AB using center D and scale factor $\frac{1}{2}$.



Solution



8

from Unit 3, Lesson 3



Student Task Statement

A polygon has perimeter 12. It is dilated with a scale factor of k , and the resulting image has a perimeter of 8. What is the scale factor?

- A. $\frac{1}{2}$
- B. $\frac{2}{3}$
- C. $\frac{3}{4}$
- D. $\frac{4}{3}$

Solution

B

9

from Unit 2, Lesson 13



Student Task Statement

Select **all** the statements that *must* be true.

- A. Parallelograms have four congruent sides.



- 
- B. Both sets of opposite sides of a parallelogram are parallel and congruent.
 - C. A trapezoid is a parallelogram.
 - D. Diagonals of a parallelogram bisect each other.
 - E. Diagonals of a parallelogram are congruent.

Solution

B, D