Unit 2, Lesson 11

Finding Intersections

Let’s think about two polynomials at once.

11.1 Math Talk: When f Meets g

Find a point mentally where the graphs of the two functions intersect, if one exists.

- $f(x) = x$ and $g(x) = 3$
- $j(x) = (x + 3)(x - 3)$ and $k(x) = 0$
- $m(x) = (x + 3)(x - 3)$ and $n(x) = (x - 3)$
- $p(x) = (x + 5)(x - 5)$ and $q(x) = (x + 3)(x - 3)$

11.2 More Points of Intersection

For each pair of polynomials given, find all points of intersection of their graphs.

1. $c(x) = x^2 - 7$ and $d(x) = 2$
2. $f(x) = (x + 7)(x - 4)$ and $g(x) = x - 4$
3. $m(x) = (x + 7)(x - 4)$ and $n(x) = (2x + 5)(x - 4)$
4. $p(x) = (x + 1)(x - 8)$ and $q(x) = (x + 2)(x - 4)$

Are you ready for more?

Find all points of intersection of the graphs of the equations $p(x) = (2x + 3)(x - 5)$ and $q(x) = (x + 5)(x + 1)(x - 3)$. Use graphing technology to check your solutions.

11.3 Graphing to Find Points of Intersection

Consider the functions $p(x) = 5x^3 + 6x^2 + 4x$ and $q(x) = 5640$.

1. Use graphing technology to find a value of x that makes $p(x) = q(x)$ true.

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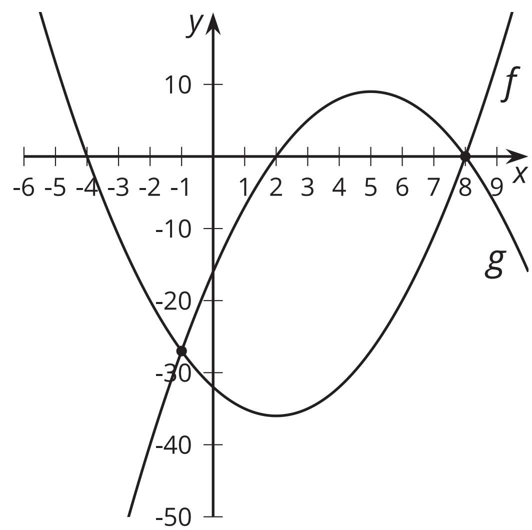
2. Using the x-value at the point of intersection, what is the value of $5x^3 + 6x^2 + 4x - 5640$?
3. What does your answer suggest is a possible factor of $5x^3 + 6x^2 + 4x - 5640$?
4. a. Write your own polynomial $m(x)$ of degree 3 or higher.
   b. Use graphing technology to estimate the values of x that make $m(x) = q(x)$ true.

Lesson 11 Summary

When asked to find all values of x that make an equation like $(x + 4)(x - 8) = (2 - x)(x - 8)$ true, one way to consider the question is to ask where the graphs of the functions $f(x) = (x + 4)(x - 8)$ and $g(x) = (2 - x)(x - 8)$ intersect.



Since the coordinate of any point of intersection has the form $(a, f(a)) = (a, g(a))$, these points must make $f(x) = g(x)$ true when $x = a$. In our example, we can tell from the graph that both $x = \text{-}1$ and $x = 8$ are solutions to the original equation.

We can also use algebra to identify solutions to $(x + 4)(x - 8) = (2 - x)(x - 8)$ by rearranging and then recognizing that both parts have a factor of $(x - 8)$ in common:

$$\begin{aligned}(x + 4)(x - 8) &= (2 - x)(x - 8) \\ (x + 4)(x - 8) - (2 - x)(x - 8) &= 0 \\ (x - 8)(x + 4 - 2 + x) &= 0 \\ (x - 8)(2x + 2) &= 0 \\ x &= 8, \text{-}1\end{aligned}$$

For polynomials created to model specific situations that have a more complicated structure, solving without using technology can be challenging, especially because the graphs of two polynomials can intersect at multiple points.