

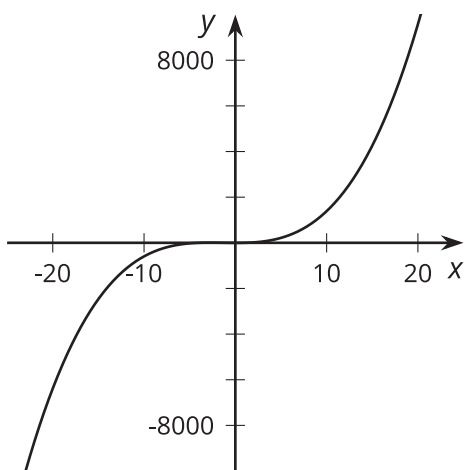
# End Behavior (Part 1)

Let's investigate the shape of polynomials.

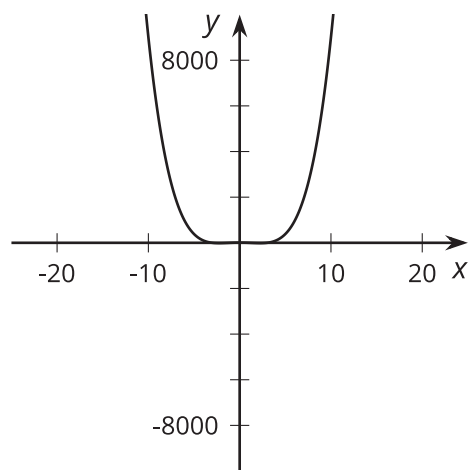
## 8.1 Notice and Wonder: A Different View

What do you notice? What do you wonder?

$$y = x^3 + 4x^2 - x - 4$$



$$y = x^4 - 10x^2 + 9$$



8.2

Polynomial End Behavior

1. In the column for your assigned polynomial, evaluate for the different values of  $x$ . Discuss what you notice with your group.

| $x$   | $y = x^2 + 1$ | $y = x^3 + 1$ | $y = x^4 + 1$ | $y = x^5 + 1$ |
|-------|---------------|---------------|---------------|---------------|
| -1000 |               |               |               |               |
| -100  |               |               |               |               |
| -10   |               |               |               |               |
| -1    |               |               |               |               |
| 1     |               |               |               |               |
| 10    |               |               |               |               |
| 100   |               |               |               |               |
| 1000  |               |               |               |               |

2. Sketch what you think the **end behavior** of your polynomial looks like, then check your work using graphing technology.



### Are you ready for more?

Mai is studying the function  $p(x) = -\frac{1}{100}x^3 + 25,422x^2 + 8x + 26$ . She makes a table of values for  $p$  with  $x = \pm 1, \pm 5, \pm 10, \pm 20$  and thinks that this function has large positive output values in both directions on the  $x$ -axis. Do you agree with Mai? Explain your reasoning.

## 8.3 Two Polynomial Equations

Consider the polynomial  $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$ .

1. Identify the degree of the polynomial.
2. Which of the 6 terms,  $2x^5$ ,  $5x^4$ ,  $30x^3$ ,  $5x^2$ ,  $88x$ , or  $60$ , is greatest when:
  - a.  $x = 0$
  - b.  $x = 1$
  - c.  $x = 3$
  - d.  $x = 5$
3. Describe the end behavior of the polynomial.

### Lesson 8 Summary

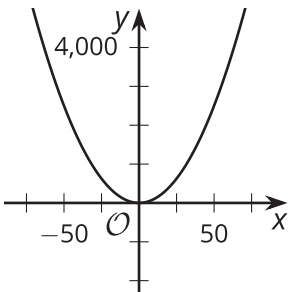
The value of the leading term determines the **end behavior** of the function, that is, how the outputs of the function change as we look at input values farther and farther from 0.

Consider the polynomial  $P(x) = x^4 - 30x^3 - 20x^2 + 1000$ . The leading term,  $x^4$ , almost seems smaller than the other three terms, and for certain values of  $x$ , this is even true. But, for values of  $x$  far away from 0, the leading term will always have the greatest value. In the case of  $P$ , as  $x$  gets larger and larger in the positive and negative directions, the output of the function gets larger and larger in the positive direction.

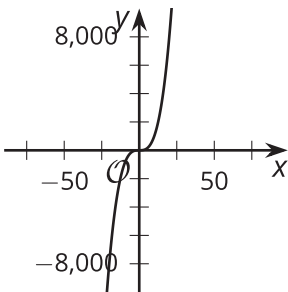
| $x$  | $x^4$          | $-30x^3$       | $-20x^2$   | 1000  | $P(x)$         |
|------|----------------|----------------|------------|-------|----------------|
| -500 | 62,500,000,000 | 3,750,000,000  | -5,000,000 | 1,000 | 66,245,001,000 |
| -100 | 100,000,000    | 30,000,000     | -200,000   | 1,000 | 129,801,000    |
| -10  | 10,000         | 30,000         | -2,000     | 1,000 | 39,000         |
| 0    | 0              | 0              | 0          | 1,000 | 1000           |
| 10   | 10,000         | -30,000        | -2,000     | 1,000 | -21,000        |
| 100  | 100,000,000    | -30,000,000    | -200,000   | 1,000 | 69,801,000     |
| 500  | 62,500,000,000 | -3,750,000,000 | -5,000,000 | 1,000 | 58,745,001,000 |

If we graph  $y = x^2$ ,  $y = x^3$  and  $y = x^4$  and zoom out, we see the following:

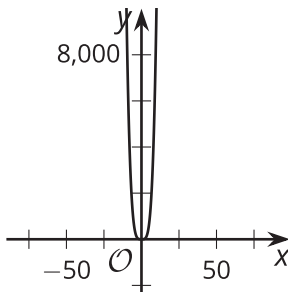
$y = x^2$



$y = x^3$



$y = x^4$



For both  $y = x^2$  and  $y = x^4$ , large positive values of  $x$  or large negative values of  $x$  each result in large positive values of  $y$ .

But for  $y = x^3$ , large positive values of  $x$  result in large positive values of  $y$ , while large negative values of  $x$  result in large negative values of  $y$ .