Unit 2, Lesson 1

Congruent Parts, Part 1

Goals

- Determine whether or not figures are congruent by reasoning about rigid transformations (in writing).
- Generate (orally and in writing) and comprehend congruence statements that establish corresponding parts.

Learning Targets

- I can identify corresponding parts from a congruence statement.
- I can use rigid transformations to figure out if figures are congruent.
- I can write a congruence statement.

Lesson Narrative

In this lesson, students recall that in a transformation, a part of the first figure and its image in the second figure are called **corresponding parts**. This lesson re-introduces students to the concept that if two figures are congruent, then each pair of corresponding parts of the figures are also congruent. Students begin applying this concept in the next activity when they make and justify conjectures about a quadrilateral they form by transforming a triangle. Constructing a viable argument includes both writing conjectures and providing reasons to support that claim (MP3).

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

Standards

Building On	**8.G.A.1, 8.G.A.1.b, 8.G.A.5**
Addressing	**HSG-CO.A.5, HSG-CO.B.6**
Building Toward	**HSG-CO.B.7, HSG-CO.B.8, HSG-CO.C.11**

Instructional Routines

- Draw It
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Preparation

Activity 2:

Students will continue adding to their reference chart in this activity. Be prepared to add to the class display. The Blank Reference Chart for students and a teacher copy of a completed version are available in the black line masters for the unit.

If there are multiple sections of this course in the same classroom, consider hiding entries on the class reference chart and revealing them at the appropriate time rather than making multiple displays.

Student Facing Learning Goals

Let’s figure out what the corresponding sides and angles in figures have to do with congruence.

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1.1 Find the Missing Angle Measures

Warm-up

10 min

Activity Narrative

In preparation for subsequent activities, this *Warm-up* reminds students about the names and relationships of angle pairs formed by parallel lines and a transversal. In particular, they are reminded that alternate interior angles formed by parallel lines and a transversal are congruent.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance students in developing their mathematical language.

Standards

Building On	**8.G.A.5**
Building Toward	**HSG-CO.B.8**

Instructional Routines

- MLR2: Collect and Display

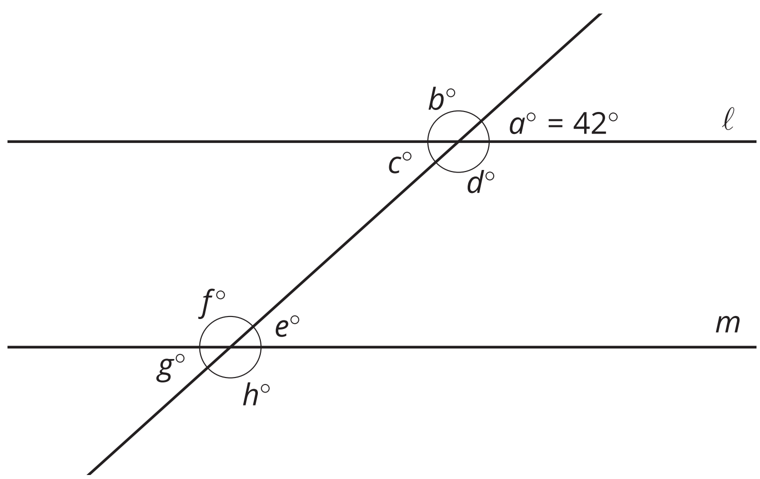
Launch

Use *Collect and Display* to create a shared reference using students' developing mathematical language. Collect the language that students use to justify the measurements for each angle. Display words and phrases, such as "congruent," "supplementary," "linear pair," "vertical angles," "corresponding angles," and "alternate interior angles."

Student Task Statement

Lines ℓ and m are parallel. $a = 42$. Find b, c, d, e, f, g, and h.

$\ell \parallel m$



Student Response

$b = 138, c = 42, d = 138, e = 42, f = 138, g = 42, h = 138$

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*.

Ask students about their reasoning for the relationship between different pairs of angles in this order:

- a and b (the angles form a linear pair)
- a and c (the angles are vertical angles)
- a and e (the angles are corresponding angles)
- c and e (the angles are alternate interior angles)

Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond.

If students describe how they found c and e by referring back to them both being equal to a, first congratulate them on an accurate use of the transitive property, then ask students if they recall the name of the angle pair that the angles with measures c and e form (alternate interior angles). Having a single statement about the angles with measure c and e makes later proofs that rely on their relationship much simpler to describe.

1.2 If We Know This, Then We Know That

15 min

Activity Narrative

The main goal of this activity is to establish that, in congruent triangles, corresponding parts must also be congruent. This theorem is intentionally not shortened to CPCTC, as students may forget what the abbreviation means if they are not reading and saying the words each time. Students will justify this theorem by recognizing that the same transformation that is used to show the triangles' congruence can also be used to show the congruence of the parts.

This activity includes the first new addition to the reference chart in this unit. Students should continue using the reference chart from a previous unit, and this statement should be added in the next blank space.

More problems like the extension can be found online by searching for "congruent halves."

Standards

Addressing	**HSG-CO.A.5, HSG-CO.B.6**
Building Toward	**HSG-CO.B.7**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Start the *Activity Synthesis* as soon as students have had a chance to think about all the questions. They will have the opportunity to formalize their language and arguments during the discussion.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "Explain how you know those segments are congruent." Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen

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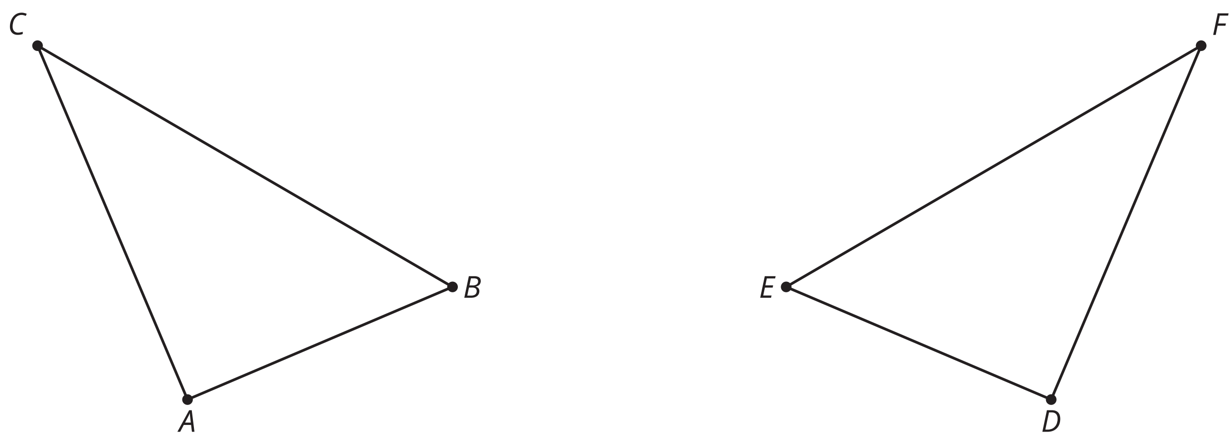
their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive. *Advances: Writing, Speaking, Listening*

Student Task Statement

Triangle ABC is congruent to triangle DEF.

$\triangle ABC \cong \triangle DEF$



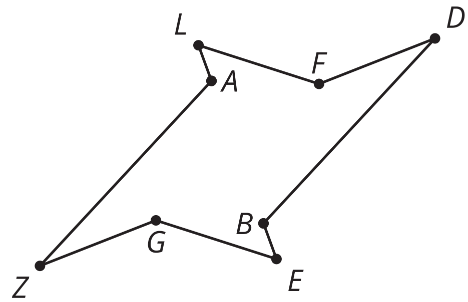
1. Find a sequence of rigid motions that takes triangle ABC to triangle DEF.
2. What is the image of segment BC after that transformation?
3. Explain how you know those segments are congruent.
4. Justify that angle ABC is congruent to angle DEF.

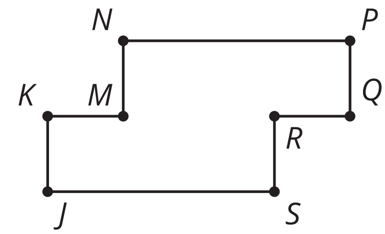
Student Response

1. Sample responses:
   - Translate from B to E. Rotate around E by angle $A'ED$. Reflect across ED.
   - Reflect triangle ABC across the perpendicular bisector of segment BE.
2. Segment EF
3. Sample response: The definition of congruent is that there exists a rigid transformation taking one figure to the other. EF is the image of BC, so they must be congruent.
4. Sample response: The definition of congruent is that there exists a rigid transformation taking one figure to the other. The sequence of rigid motions that takes triangle ABC to triangle DEF also takes angle ABC to angle DEF, so they are congruent.

Are You Ready for More?

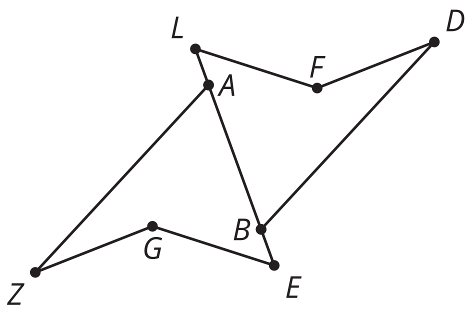
For each figure, draw additional line segments to divide the figure into 2 congruent polygons. Label any new vertices, and identify the corresponding vertices of the congruent polygons.



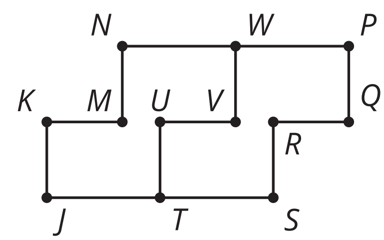


Extension Student Response

Sample responses:



$AZGE$ corresponds to $BDFL$



$JTUVWNMK$ corresponds to $TSRQPWVU$

Activity Synthesis

Invite students to share how they know segments BC and EF are congruent.

If students say that the triangles are congruent, so the segments must be congruent, tell them that is what they are about to prove. Continue the discussion until students understand that segment BC is congruent to segment EF because we know we can use rigid transformations to line up the triangles. That means we know segments BC and EF will be lined up when the triangles are lined up, so the segments must be congruent.

Arrange students in groups of 2. Invite them to make this specific argument more general—that is, turn the example into a proof of "If two figures are congruent, then corresponding segments of those figures must be congruent." Give students 1 minute of quiet think time followed by 2 minutes to write an outline of the proof with their partner. Then invite pairs to share parts of their proof until the class has a complete proof. Here is a sample proof:

1. If figures are congruent, there is a rigid transformation that takes one figure to the other.
2. If there is a rigid transformation that takes one figure to another, it also takes one segment of the figure to a segment of the other, and we call these two "corresponding segments."
3. Since there is a rigid transformation taking one segment to another, those segments are congruent.
4. Therefore, if figures are congruent, then the corresponding segments of those figures must also be congruent.

Ask, "Does this argument work for angles?" (Yes, if you replace the word "segment" with the word "angle" throughout the proof.)

Add the following theorem to the class reference chart, and ask students to add it to their reference charts:

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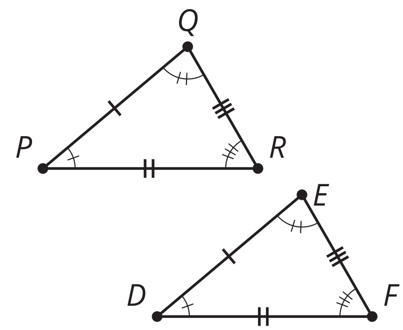
If two figures are congruent, then **corresponding parts** of those figures must be congruent. (Theorem)

$\triangle DEF \cong \triangle PQR$ so $\overline{PQ} \cong \overline{DE}, \overline{PR} \cong \overline{DF}, \overline{QR} \cong \overline{EF}$, $\angle P \cong \angle D, \angle Q \cong \angle E, \angle R \cong \angle F$



Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, color code corresponding parts.
Supports accessibility for: Visual-Spatial Processing

1.3 Making Quadrilaterals

10 min

Activity Narrative

The purpose of this activity is to have students identify corresponding parts in congruent triangles.

Monitor for students who:

- Make arguments based on properties of rotations.
- Make arguments based on alternate interior angles being congruent.

Making dynamic geometry software available gives students an opportunity to choose appropriate tools strategically (MP5).

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Building On	**8.G.A.1**
Addressing	**HSG-CO.A.5**
Building Toward	**HSG-CO.C.11**

Instructional Routines

- Draw It
- MLR7: Compare and Connect
- MLR8: Discussion Supports

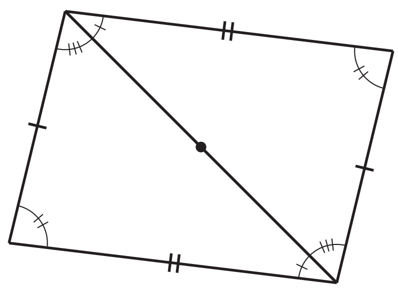
Launch

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

1. Draw a triangle.
2. Find the midpoint of the longest side of your triangle.
3. Rotate your triangle 180° using the midpoint of the longest side as the center of the rotation.
4. Identify the **corresponding parts**, and mark which segments and angles must be congruent.
5. Make a conjecture and justify it.
    a. What type of quadrilateral have you formed?
    b. What is the definition of that quadrilateral type?
    c. Why must the quadrilateral you have fit the definition?

Student Response



1.
2. See image.
3. See image.
4. See image.
5. Sample responses:
    a. The quadrilateral is a parallelogram.
    b. The opposite sides must be parallel.
    c. The opposite sides are parallel because rotation by 180° takes lines to parallel lines, so the opposite sides in my quadrilateral are parallel.
    By corresponding parts, the alternate interior angles are congruent, so the lines are parallel.

Building on Student Thinking

If students are struggling to perform the rotation, encourage them to use whatever tools they are most comfortable with, including sketching.

Activity Synthesis

The purpose of this discussion is to share examples of convincing arguments.

Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- "What did the approaches have in common? How were they different?"
- "Which theorems or definitions in your reference chart does each approach rely on?"

If no student used corresponding parts to establish alternate interior angles as congruent, ask students to look for structure by looking at the congruence marks and seeing whether they match any diagrams on the reference chart.

Access for English Language Learners

MLR8 Discussion Supports. For each justification that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Lesson Synthesis

Display the image from the *Warm-up* again along with the statement "Triangle $A'B'C'$ is a reflection of triangle ABC across line BC." Arrange students in groups of 2. Invite groups to choose a pair of corresponding parts and write a justification why those parts are congruent.

1.4 Making Angle Bisectors

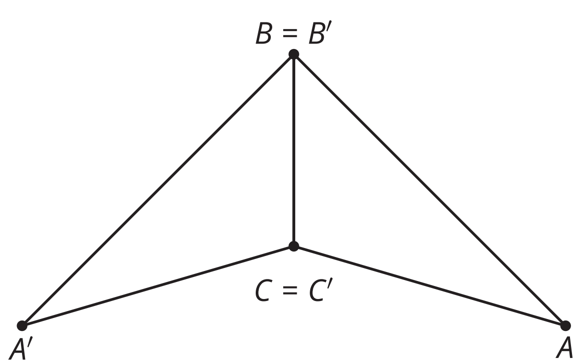
Cool-down

5 min

Standards

Building On	**8.G.A.1.b**
Addressing	**HSG-CO.B.6**
Building Toward	**HSG-CO.B.7**

Student Task Statement



Triangle $A'B'C'$ is a reflection of triangle ABC across line BC. Explain how you know that ray BC is the angle bisector of angle ABA'.

Student Response

Sample responses:

- Triangle ABC is congruent to triangle $A'B'C'$ since they are reflections of each other. That means angles $A'B'C'$ and ABC must be congruent because they are corresponding parts of congruent triangles. So BC must be the angle bisector of ABA' since it splits the angle into two congruent angles.
- Angle $A'B'C'$ is a reflection of angle ABC across BC because it is part of the triangle that was reflected. That means angles $A'B'C'$ and ABC must be congruent. So BC must be the angle bisector of ABA' since it splits the angle into two congruent angles.

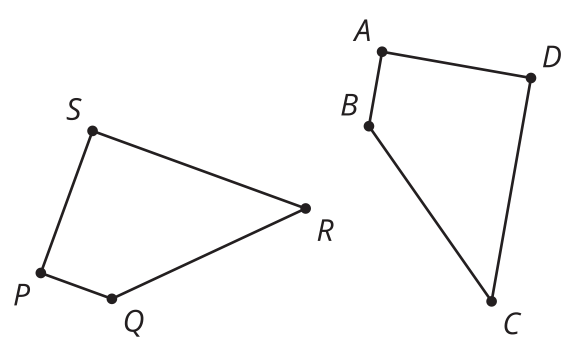
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 1 Summary

If a part of the image matches up with a part of the original figure, we call them **corresponding parts**. The part could be an angle, point, or side. We can find corresponding angles, corresponding points, or corresponding sides.

If two figures are congruent, then there is a rigid transformation that takes one figure onto the other. The same rigid transformation can also be applied to individual parts of the figure, such as segments and angles, because rigid transformations act on every point on the plane. Therefore, the corresponding parts of two congruent figures are congruent to each other.



Using a translation and a rotation we can take quadrilateral $ABCD$ to quadrilateral $PQRS$. Now that we know the two figures are congruent, we also know that all the corresponding parts are congruent. Each of these statements (and more!) must be true:

- Angle P is congruent to angle A.
- Segment BC is congruent to segment QR.
- Angle D is congruent to angle S.
- Segment PS is congruent to segment AD.

Glossary

- corresponding parts

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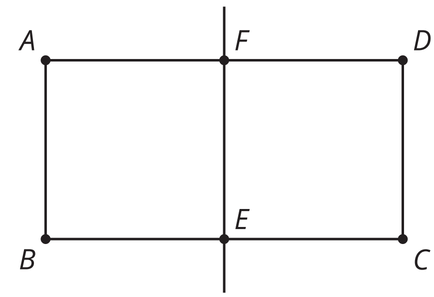
Lesson 1 Practice Problems

1 Student Task Statement

When rectangle $ABCD$ is reflected across line EF, the image is $DCBA$. How do you know that segment AB is congruent to segment DC?



A. A rectangle has 2 pairs of parallel sides.

B. Any 2 sides of a rectangle are congruent.

C. Congruent parts of congruent figures are corresponding.

D. Corresponding parts of congruent figures are congruent.

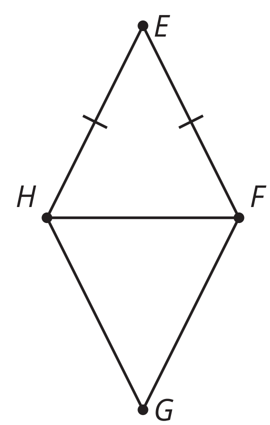
Solution

D

2 Student Task Statement

Triangle FGH is the image of isosceles triangle FEH after a reflection across line HF. Select **all** the statements that are a result of corresponding parts of congruent triangles being congruent.

$\overline{FE} \cong \overline{HE}$



A. $EFGH$ is a rectangle.

B. $EFGH$ has 4 congruent sides.

C. Diagonal FH bisects angles EFG and EHG.

D. Diagonal FH is perpendicular to side FE.

E. Angle FEH is congruent to angle FGH.

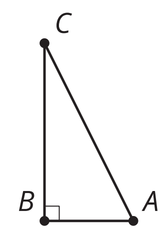
Solution

B, C, E

3

Student Task Statement

Reflect right triangle ABC across line BC. Classify triangle ACA' according to its side lengths. Explain how you know.



Solution

Sample response: Triangle ACA' must be isosceles. Since a reflection preserves distance, side AC is congruent to side CA'. Since at least two sides are congruent, triangle ACA' is isosceles.

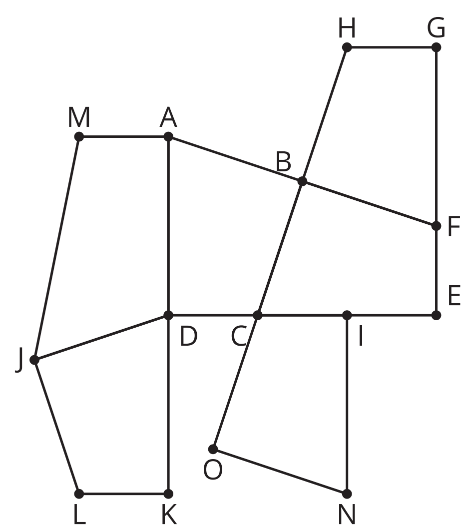
4

from Unit 1, Lesson 18

Student Task Statement

a. Identify a figure that is the result of a rigid transformation of quadrilateral $ABCD$.

b. Describe a rigid transformation that would take $ABCD$ to that figure.



Solution

Sample response:

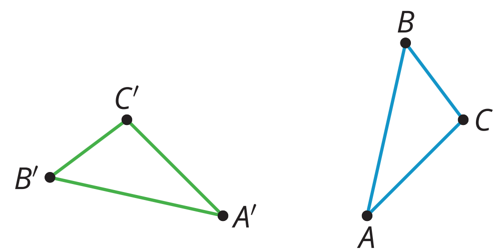
a. $FBHG$
b. Rotate $ABCD$ using center B clockwise 180 degrees.

Quadrilateral $MADJ$ is the only one that cannot be described as a sequence of transformations of $ABCD$.

5 from Unit 1, Lesson 17

Student Task Statement

Triangle ABC is congruent to triangle $A'B'C'$. Describe a sequence of rigid motions that takes A to A', B to B', and C to C'.



Solution

Sample response: Translate triangle ABC by the directed line segment AA'. Rotate the image using center A' until the image of B coincides with B'.