

# What Is a Logarithm?

Let's learn about logarithms.

## 9.1 Math Talk: Finding Solutions

Find or estimate the value of each variable mentally.

- $4^a = 16$

- $4^b = 2$

- $4^{\frac{5}{2}} = c$

- $4^d = 56$

## 9.2 A Table of Numbers

| $x$ | $\log_{10}(x)$ |
|-----|----------------|
| 2   | 0.3010         |
| 3   | 0.4771         |
| 4   | 0.6021         |
| 5   | 0.6990         |
| 6   | 0.7782         |
| 7   | 0.8451         |
| 8   | 0.9031         |
| 9   | 0.9542         |
| 10  | 1              |

| $x$ | $\log_{10}(x)$ |
|-----|----------------|
| 20  | 1.3010         |
| 30  | 1.4771         |
| 40  | 1.6021         |
| 50  | 1.6990         |
| 60  | 1.7782         |
| 70  | 1.8451         |
| 80  | 1.9031         |
| 90  | 1.9542         |
| 100 | 2              |

| $x$   | $\log_{10}(x)$ |
|-------|----------------|
| 200   | 2.3010         |
| 300   | 2.4771         |
| 400   | 2.6021         |
| 500   | 2.6990         |
| 600   | 2.7782         |
| 700   | 2.8451         |
| 800   | 2.9031         |
| 900   | 2.9542         |
| 1,000 | 3              |

| $x$    | $\log_{10}(x)$ |
|--------|----------------|
| 2,000  | 3.3010         |
| 3,000  | 3.4771         |
| 4,000  | 3.6021         |
| 5,000  | 3.6990         |
| 6,000  | 3.7782         |
| 7,000  | 3.8451         |
| 8,000  | 3.9031         |
| 9,000  | 3.9542         |
| 10,000 | 4              |

1. Analyze the table and discuss with a partner what you think the table tells us.
2. Use the table to find the value of the unknown exponent that makes each equation true.
  - a.  $10^w = 1,000$
  - b.  $10^y = 9$
  - c.  $10^z = 90$
3. Notice that some values in the columns labeled  $\log_{10} x$  are whole numbers, but most are decimals. Why do you think that is?

## 9.3

## Hello, Logarithm!

1. Here are two true equations based on the information from the table:

$$\log_{10} 100 = 2$$

$$\log_{10} 1,000 = 3$$

What values could replace the “?” in these equations to make them true?

- $\log_{10} 1,000,000 = ?$
  - $\log_{10} 1 = ?$
  - $\log_{10} ? = 7$
2. Estimate the value of  $\log_{10} 610$ ? Be prepared to explain how you know.
3. The term *log* is short for **logarithm**. Discuss the following questions with a partner, and record your responses:
- What do you think logarithm means or does?
  - Next to “log” is a subscript—a number or letter printed smaller and below the line of text—that is called the “base.” What do you think the base of the logarithm tells us?
  - What about the other two numbers on either side of the equal sign (for example, the 100 and the 2 in  $\log_{10} 100 = 2$ )? What do they tell us?

**Are you ready for more?**

- Let  $x$  be a number. For which  $x$  is  $\log_{10}(x)$  an integer?
- Use the table to write an approximate value for  $\log_{10}(900,000)$ . Explain your reasoning.

## Lesson 9 Summary

We know how to solve equations such as  $10^a = 10,000$  or  $10^b = \frac{1}{100}$  by thinking about integer powers of 10. The solutions are  $a = 4$  and  $b = -2$ . What about an equation such as  $10^p = 250$ ?

Because  $10^2 = 100$  and  $10^3 = 1,000$ , we know that  $p$  is between 2 and 3. We can use a **logarithm** to represent the exact solution to this equation and write it as:

$$p = \log_{10} 250$$

The expression  $\log_{10} 250$  is read “the log, base 10, of 250.”

- The small, slightly lowered “10” refers to the base of 10.
- The 250 is the value of the power of 10.
- $\log_{10} 250$  is the value of the exponent  $p$  that makes  $10^p$  equal 250.

In the specific case where the base of the logarithm is 10, the “log” can be written without the number 10. For example,  $\log_{10} 250$  can also be written as  $\log 250$ , and this expression is read “the log of 250.”

One way to estimate logarithms is with a logarithm table. For example, using this base 10 logarithm table we can see that  $\log_{10} 250$  is between 2.3010 and 2.4771.

| $x$ | $\log_{10}(x)$ |
|-----|----------------|
| 2   | 0.3010         |
| 3   | 0.4771         |
| 4   | 0.6021         |
| 5   | 0.6990         |
| 6   | 0.7782         |
| 7   | 0.8451         |
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| 60  | 1.7782         |
| 70  | 1.8451         |
| 80  | 1.9031         |
| 90  | 1.9542         |
| 100 | 2              |

| $x$   | $\log_{10}(x)$ |
|-------|----------------|
| 200   | 2.3010         |
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| 400   | 2.6021         |
| 500   | 2.6990         |
| 600   | 2.7782         |
| 700   | 2.8451         |
| 800   | 2.9031         |
| 900   | 2.9542         |
| 1,000 | 3              |

| $x$    | $\log_{10}(x)$ |
|--------|----------------|
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| 3,000  | 3.4771         |
| 4,000  | 3.6021         |
| 5,000  | 3.6990         |
| 6,000  | 3.7782         |
| 7,000  | 3.8451         |
| 8,000  | 3.9031         |
| 9,000  | 3.9542         |
| 10,000 | 4              |