Compounding Interest

Let's explore different ways of repeatedly applying a percent increase.

16.1 Five Years Later

You owe 12% interest each year on a $500 loan. If you make no payments and take no additional loans, what will the loan balance be after 5 years?

Write an expression to represent the balance and evaluate it to find the answer in dollars.

16.2 Resizing Images

Andre and Mai need to enlarge two images for a group project. The two images are the same size when they begin.

Andre makes a scaled copy of his image, increasing the lengths by 10%. It is still a little too small, so he increases the lengths by 10% again.

Mai says, “If I scale my image and increase the lengths by 20%, our images will be exactly the same size.”

Do you agree with Mai? Explain or show your reasoning.

Are you ready for more?

What if you want to calculate $5 \cdot (1.01)^{10}$ but don't have a calculator? As long as the value you're multiplying is close to 1, you can estimate using addition rather than multiplication. $5 + (0.01) \cdot 10 = 5.1$ which is quite close to the actual value of the original expression, which is approximately 5.523.

- The quantity $(1.02)^7$ is hard to calculate by hand. Use mental math to compute $1 + (0.02) \cdot 7$ to get a good approximation to it.
- Estimate $(0.99)^9$. Use a calculator to compare your estimate to the actual value.
- Estimate $(1.6)^{11}$. Use a calculator to compare your estimate to the actual value. What is different about this example?

16.3 Earning Interest

A bank account has a monthly interest rate of 1% and initial balance of $1,000. Any earned interest is added to the account, and no other deposits or withdrawals are made.

1. What is the account balance after 6 months, 1 year, 2 years, and 5 years? Show your reasoning.

2. Write an equation expressing the account balance, a, in terms of the number of months, m. Assume that all interest earned continues to be added to the account and that no other deposits or withdrawals are made.

3. How much interest will the account earn in 1 year? What percentage of the initial balance is that? Show your reasoning.

4. The term annual return refers to the percent of interest an account holder could expect to receive in one year. Discuss with your partner: If you were the bank, would you advertise the account as having a 12% annual return? Why or why not? Use your work so far to explain your reasoning.

Lesson 16 Summary

Suppose a runner runs 4 miles a day this month. She is increasing her daily running distance by 25% next month, and then by 25% of that the month after. Will she be running 50% more than her current daily distance two months from now?

It is tempting to think that two months from now she will be running 6 miles, because twice 25% is 50%, and 50% more than her current daily distance is $4 \cdot (1.5)$. But if we calculate the increase one month at a time, we can see that next month she will run $4 \cdot (1.25)$ or 5 miles. The month after that she will run $5 \cdot (1.25)$ or 6.25 miles.

So two months from now her daily distance will actually be:

$$4 \cdot (1.25)^2$$

Two repeated 25% increases actually lead to an overall increase of 56.25% rather than of 50%, because $1.25^2 = 1.5625$. Applying a percent increase on an amount that has had a prior percent increase is called *compounding.*

Compounding happens when we calculate interest on money in a bank account or on a loan. An account that earns 2% interest every month does not actually earn 24% a year because the interest applies not only to the original amount, but also to any interest already included earlier. Let's say a savings account has $300 and no other deposits or withdrawals are made. The account balances after some months are shown in this table.

number of months	account balance in dollars
1	$300 \cdot (1.02)$
2	$300 \cdot (1.02)^2$
3	$300 \cdot (1.02)^3$
12	$300 \cdot (1.02)^{12}$

$(1.02)^{12} \approx 1.2682$, so the account will grow by about 26.82% in one year. This rate is called the *effective interest rate*. It reflects how the account balance actually changes after one year.

The 24% is called the *nominal interest rate.* It is the stated or published rate and is usually used to determine the monthly, weekly, or daily rates (if interest were to be calculated at those intervals).