More Arithmetic with Complex Numbers

Let's practice adding, subtracting, and multiplying complex numbers.

11.1   Which Three Go Together: Complex Expressions

Which three go together? Why do they go together?

A

$$2i \cdot i$$

B

$$(1 + i) + (1 - i)$$

C

$$(1 + i)^2$$

D

$$(1 + 2i)(1 - 2i)$$

11.2   Powers of i

1. Write each power of i in the form $a + bi$, where a and b are real numbers. If a or b is zero, you do not need to write that part of the number. For example, $0 + 3i$ can be expressed as $3i$.

i^0 i^1 i^2

i^3 i^4 i^5

i^6 i^7 i^8

2. Use any patterns you noticed to rewrite i^{100} in a similar way. Explain your reasoning.

3. Use any patterns you noticed to rewrite i^{38} in a similar way. Explain your reasoning.

💡 **Are you ready for more?**

1. Write each power of $1 + i$ in the form $a + bi$, where a and b are real numbers. If a or b is zero, you do not need to write that part of the number. For example, $0 + 3i$ can be expressed as $3i$.

   $(1 + i)^0$ $(1 + i)^1$ $(1 + i)^2$

   $(1 + i)^3$ $(1 + i)^4$ $(1 + i)^5$

   $(1 + i)^6$ $(1 + i)^7$ $(1 + i)^8$

2. Compare and contrast the powers of $1 + i$ with the powers of i when plotted on the complex plane. What is the same? What is different?

For each row, your partner and you will each rewrite an expression so it has the form $a + bi$, where a and b are real numbers. You and your partner should get the same answer. If you disagree, work to reach an agreement.

partner A	partner B
$(7 + 9i) + (3 - 4i)$	$5i(1 - 2i)$
$2i(3 + 4i)$	$(1 + 2i) - (9 - 4i)$
$(4 - 3i)(4 + 3i)$	$(5 + i) + (20 - i)$
$(2i)^4$	$(3 + i\sqrt{7})(3 - i\sqrt{7})$
$(1 + i\sqrt{5}) - (-7 - i\sqrt{5})$	$(-2i)(-\sqrt{5} + 4i)$
$\left(\frac{1}{2}i\right)\left(\frac{1}{3}i\right)\left(\frac{3}{4}i\right)$	$\left(\frac{1}{2}i\right)^3$

👥 Lesson 11 Summary

Suppose we want to write the product $(3 + 5i)(7 - 2i)$ in the form $a + bi$, where a and b are real numbers. For example, we might want to compare our solution with a partner's, and having answers in the same form makes that easier. Using the distributive property,

$$\begin{aligned}
(3 + 5i)(7 - 2i) &= 21 - 6i + 35i - 10i^2 \\
&= 21 + 29i - 10(\text{-}1) \\
&= 21 + 29i + 10 \\
&= 31 + 29i
\end{aligned}$$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that $i^2 = \text{-}1$ with the fact that $\text{-}i^2 = \text{-}(\text{-}1) = 1$.

Next, suppose we want to write the difference $(\text{-}6 + 3i) - (2 - 4i)$ as a single complex number in the form $a + bi$. Distributing the negative and combining like terms, we get:

$$\begin{aligned}
(\text{-}6 + 3i) - (2 - 4i) &= \text{-}6 + 3i - 2 - (\text{-}4i) \\
&= \text{-}8 + 3i + 4i \\
&= \text{-}8 + 7i
\end{aligned}$$

Again, it is important to be precise with negative signs. It is a common mistake to subtract $4i$ rather than subtracting $\text{-}4i$.