Unit 2, Lesson 11

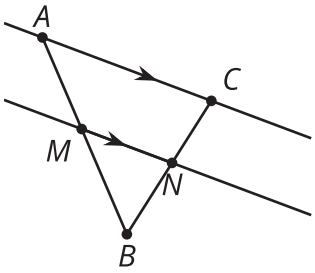
# Splitting Triangle Sides with Dilation (Part 2)

Let's investigate parallel segments in triangles.

## 11.1 Notice and Wonder: Parallel Segments

What do you notice? What do you wonder?

$\overleftrightarrow{AC} \parallel \overleftrightarrow{MN}$



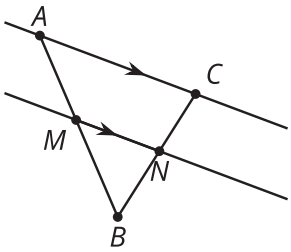

## 11.2 Prove It: Parallel Segments

Does a line parallel to one side of a triangle always create similar triangles?

1. Create several examples. Decide if the conjecture is true or false. If it's false, make a more specific true conjecture.

2. Find any additional information that you can be sure is true. Label it on the diagram.

$\overleftrightarrow{AC} \parallel \overleftrightarrow{MN}$

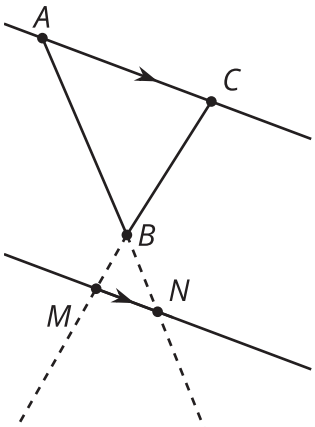



3. Write an argument that would convince a skeptic that your conjecture is true.

## Are you ready for more?

If the line parallel to one side of the triangle does not intersect the other sides of the triangle, does it still create a similar triangle if the sides of the original triangle are extended? Modify your reasoning from this activity to show that triangle $ABC$ is similar to triangle $NBM$.

$\overleftrightarrow{AC} \parallel \overleftrightarrow{NM}$

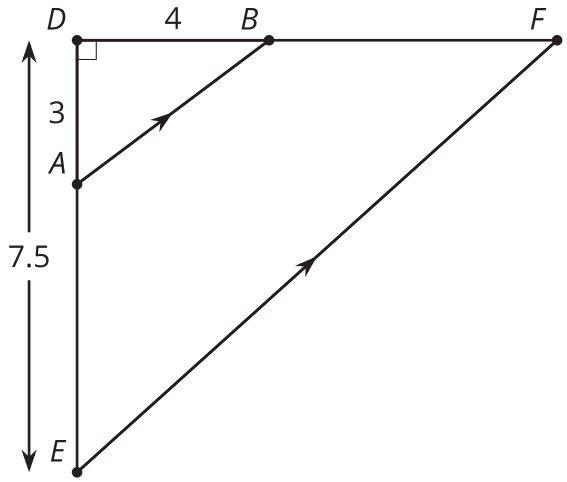


# 11.3 Preponderance of Proportional Relationships

Find the length of each unlabelled side.

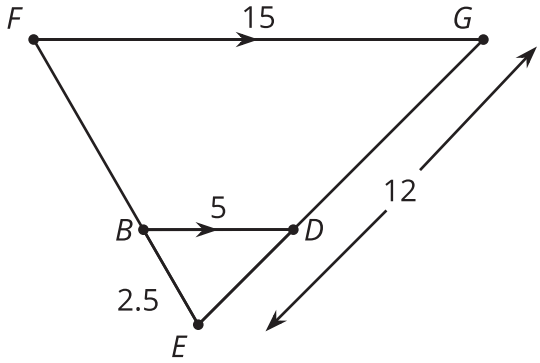
1. Segments $AB$ and $EF$ are parallel.
   - $AB =$
   - $DF =$
   - $EF =$

$\overline{AB} \parallel \overline{EF}, \overline{AD} \perp \overline{DB}$



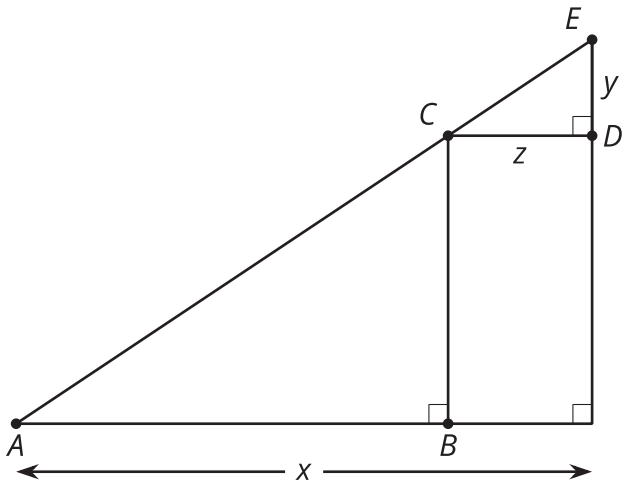

2. Segments $BD$ and $FG$ are parallel. Segment $EG$ is 12 units long. Segment $EB$ is 2.5 units long.
   - $EF =$
   - $ED =$

$\overline{BD} \parallel \overline{FG}$



## Are you ready for more?

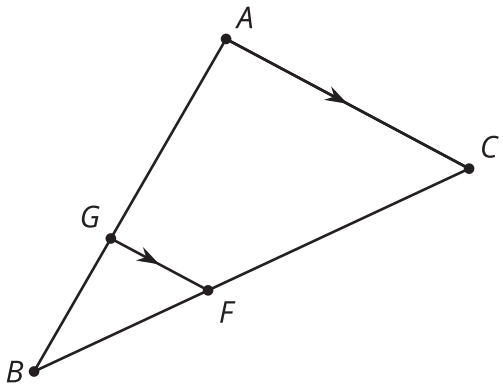
Find the lengths of sides $CE$, $CB$, and $CA$ in terms of $x$, $y$, and $z$. Explain or show your reasoning.




## Lesson 11 Summary

In triangle $ABC$, segment $GF$ is parallel to segment $AC$. We can show that corresponding angles in triangle $ACB$ and triangle $GFB$ are congruent, so the triangles are similar by the Angle-Angle Triangle Similarity Theorem. There must be a dilation that sends triangle $GFB$ to triangle $ACB$, and so pairs of corresponding side lengths are in the same proportion. Then we can show that segment $GF$ divides segments $AB$ and $CB$ proportionally. In other words, $\frac{BG}{GA} = \frac{BF}{FC}$.

$\overline{FG} \parallel \overline{AC}$




For example, suppose $G$ is $\frac{2}{3}$ of the way from $A$ to $B$ and $F$ is $\frac{2}{3}$ of the way from $C$ to $B$. Then if $BA = 9$ and $BC = 12$, we know that $GA = 6$ and $FC = 8$. What will $BG$ and $BF$ equal? Since $BG = 3$ and $BF = 4$, we know that $\frac{3}{6} = \frac{4}{8}$ and can show that $\frac{BG}{GA} = \frac{BF}{FC}$.

This argument holds in general. A segment in a triangle that is parallel to one side of the triangle divides the other two sides of the triangle proportionally.