Unit 4, Lesson 11

Using an Algorithm to Divide Fractions

Goals

- Coordinate (orally) different strategies for dividing by a fraction.
- Find the quotient of two fractions, and explain (orally, in writing, and using other representations) the solution method.
- Generalize a process for dividing a number n by a fraction $\frac{a}{b}$, and justify (orally) why it can expressed as $n \cdot \frac{b}{a}$.

Learning Targets

- I can describe and apply a rule to divide numbers by any fraction.

Lesson Narrative

In this lesson, students complete the process of developing a general algorithm for dividing a fraction by a fraction.

First, students calculate quotients using the steps they observed previously when dividing a number by a fraction—multiply the number by the denominator of the fraction and divide by the numerator. They compare the results to quotients found by reasoning with a tape diagram and verify that the two methods produce the same quotient. They sum up the steps as an algorithm: To divide a number n by $\frac{a}{b}$, we multiply n by $\frac{b}{a}$. Students learn that $\frac{b}{a}$ is the **reciprocal** of $\frac{a}{b}$ because the two numbers multiply to 1.

With an additional strategy at their disposal, students practice calculating quotients involving a wider variety of numbers (whole numbers and fractions). They begin to see the flexibility and efficiency of the algorithm, especially when it is impractical to reason about the division in terms of equal-size groups, with or without using diagrams.

Standards

Building On **5.NF.B.4**

Addressing **6.NS.A.1**

Instructional Routines

- 5 Practices
- Math Talk
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Colored pencils: Activity 2

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's divide fractions using the rule we learned.

11.1 Math Talk: Multiplying Fractions

Warm-up

5 min

Activity Narrative

This *Math Talk* focuses on products of fractions. It encourages students to interpret multiplication expressions and to rely on properties of operations and what they know about unit and non-unit fractions (including whole numbers and mixed numbers) to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students use an algorithm to divide a fraction by another fraction.

In explaining their reasoning, students need to be precise in their word choice and use of language (MP6).

Standards

Building On **5.NF.B.4**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Before moving to the next problem, use the questions in the activity synthesis to involve more students in the conversation.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each product mentally.

- $\frac{1}{8} \cdot 8$
- $\frac{1}{8} \cdot \frac{8}{3}$
- $\frac{9}{8} \cdot \frac{4}{3}$
- $1\frac{1}{8} \cdot \frac{4}{9}$

Student Response

1. 1. Sample reasoning:
   - One-eighth of 8 is 1.
   - Eight groups of $\frac{1}{8}$ make 1.
   - $\frac{1}{8}$ times 8 (or $\frac{8}{1}$) is $\frac{8}{8}$, which is 1.
2. $\frac{1}{3}$ (or equivalent). Sample reasoning:
   - The product is a third of the first product because $\frac{8}{3}$ is a third of 8 (or 8 divided by 3).
   - $\frac{1}{8} \cdot \frac{8}{3} = \frac{8}{24}$, which is equivalent to $\frac{1}{3}$.
3. $\frac{3}{2}$ (or equivalent)
   - $\frac{9}{8} \cdot \frac{4}{3} = \frac{36}{24}$, which is equivalent to $\frac{3}{2}$.
   - $\frac{9}{8}$ is 9 times $\frac{1}{8}$, and $\frac{4}{3}$ is $\frac{8}{3}$ divided by 2, so the product is 9 times the previous answer divided by 2, or $9 \cdot \frac{1}{3} \cdot \frac{1}{2}$, which is $\frac{9}{6}$ or $\frac{3}{2}$.
4. $\frac{1}{2}$ (or equivalent)
   - $1\frac{1}{8}$ is $\frac{9}{8}$, and $\frac{9}{8} \cdot \frac{4}{9} = \frac{36}{72}$, which is $\frac{1}{2}$.
   - $\frac{9}{8} \cdot \frac{4}{9}$ is equivalent to $\frac{9}{9} \cdot \frac{4}{8}$ or $(1 \cdot \frac{1}{2})$, which is $\frac{1}{2}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Highlight that the product of two fractions can be found by multiplying the numerators and multiplying the denominators.

If students mention "canceling" a numerator and a denominator that share a common factor, demonstrate using the term "dividing" instead. For example, if a student suggests that in the second expression ($\frac{1}{8} \cdot \frac{8}{3}$) the 8 in $\frac{1}{8}$ and the 8 in the $\frac{8}{3}$ "cancel out," rephrase the statement by saying that dividing the 8 in the numerator by the 8 in the denominator gives us 1, and multiplying by 1 does not change the other numerator or denominator.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because . . . ." or "I noticed ________ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

11.2 Dividing a Fraction by a Fraction

15 min

Activity Narrative

There is a digital version of this activity.

This is the final activity in a series that leads students toward a general procedure for dividing fractions. Students verify previous observations about the steps for dividing non-unit fractions (namely, multiplying by the denominator and dividing by the numerator) and contrast the results with those found using tape diagrams. They then generalize these steps as an algorithm and apply it to answer other division questions.

As students discuss in their groups, listen to their observations and explanations. Select students with clear explanations to share later.

In the digital version of the activity, students can choose to use an applet to represent division of two fractions. The applet allows students to represent the fractions on two tape diagrams and measure how many times one fraction would fit in another. Students who need support in processing abstract visual information may benefit from the ability to represent the two fractions separately and partition each tape dynamically. If students don't have individual access, displaying the applet for all to see would be helpful.

Standards

Addressing **6.NS.A.1**

Launch

Arrange students in groups of 2. Tell students that for the first question each partner will use different ways to divide a fraction by a fraction. Give students 2–3 minutes of quiet work time for the first question and time to complete the rest of the activity with their partner.

Provide access to colored pencils. Some students may find it helpful to identify whole groups and partial groups on a tape diagram by coloring.

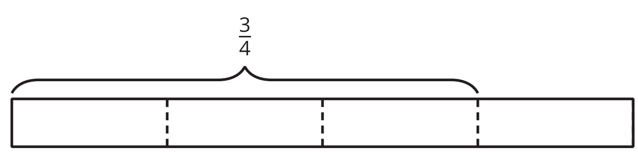
Student Task Statement

Work with a partner. One person works on the questions labeled “Partner A” and the other person works on those labeled “Partner B.”

1. Partner A: Find the value of each expression by completing the diagram.

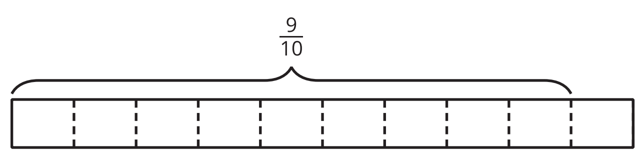
   a. $\frac{3}{4} \div \frac{1}{8}$

   How many $\frac{1}{8}$s are in $\frac{3}{4}$?

   b. $\frac{9}{10} \div \frac{3}{5}$

   How many $\frac{3}{5}$s are in $\frac{9}{10}$?

   Partner B:

Elena said, "If I want to divide 4 by $\frac{2}{5}$, I can multiply 4 by 5 and then divide it by 2 or multiply it by $\frac{1}{2}$."

Find the value of each expression using the strategy Elena described.

a. $\frac{3}{4} \div \frac{1}{8}$

b. $\frac{9}{10} \div \frac{3}{5}$

2. Discuss with your partner:

   a. Where in the diagram for $\frac{3}{4} \div \frac{1}{8}$ can we see the multiplication by the denominator 8?

   b. Where in the diagram for $\frac{9}{10} \div \frac{3}{5}$ can we see the division by the numerator 3?

   c. Where in each diagram do you see the quotient?

3. Complete this sentence based on what you noticed:

   To divide a number n by a fraction $\frac{a}{b}$, we can multiply n by ________ and then divide the product by ________.

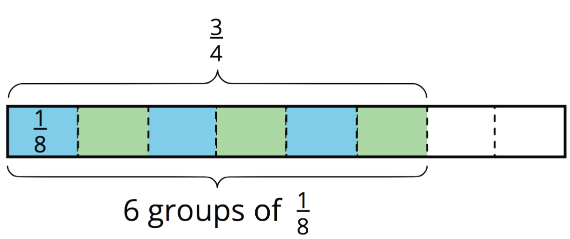
4. Select **all** the equations that represent the sentence you completed.

   - $n \div \frac{a}{b} = n \cdot b \div a$
   - $n \div \frac{a}{b} = n \cdot a \div b$
   - $n \div \frac{a}{b} = n \cdot \frac{a}{b}$
   - $n \div \frac{a}{b} = n \cdot \frac{b}{a}$

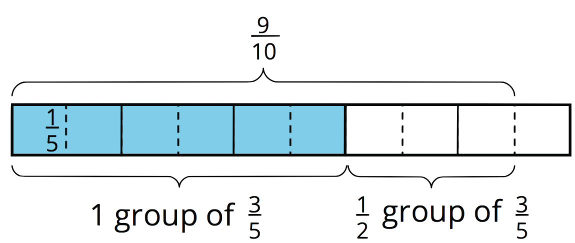
Student Response

1. Partner A

a.



b.



Partner B

a. $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot 8 = 6$

b. $\frac{9}{10} \div \frac{3}{5} = \frac{9}{10} \cdot 5 \div 3 = \frac{3}{2}$ or $1\frac{1}{2}$

2. Sample response:

   a. The length of the tape, which is 1 whole, is partitioned into eighths, creating 8 parts.

   b. The parts are put into groups of 3 fifths.

   c. In the first diagram, it's the number of $\frac{1}{8}$s in $\frac{3}{4}$. In the second diagram, it's the number of $\frac{3}{5}$s in $\frac{9}{10}$.

3. Multiply n by b and then divide the product by a.

4. $n \div \frac{a}{b} = n \cdot b \div a$ and $n \div \frac{a}{b} = n \cdot \frac{b}{a}$

Activity Synthesis

Invite a couple of students to share their conclusion about how to divide a number by any fraction. Then, review the sequence of reasoning that led us to this conclusion using both numerical examples and algebraic statements throughout. Remind students that in the past few activities, we learned that:

- Dividing by a whole number b is the same as multiplying by a unit fraction $\frac{1}{b}$.

  For instance, dividing by 5 is the same as multiplying by $\frac{1}{5}$.
- Dividing by a unit fraction $\frac{1}{b}$ is the same as multiplying by a whole number b.

  For instance, dividing by $\frac{1}{7}$ is the same as multiplying by 7.
- Dividing by a fraction $\frac{a}{b}$ is the same as multiplying by a unit fraction $\frac{1}{a}$ and multiplying by a whole number b, which is the same as multiplying by $\frac{b}{a}$.

  For instance, dividing by $\frac{5}{7}$ is the same as multiplying by $\frac{1}{5}$, and then by 7.

  Performing these two steps gives the same result as multiplying by $\frac{7}{5}$.

Tell students that pairs of numbers such as 5 and $\frac{1}{5}$, $\frac{1}{7}$ and 7, and $\frac{5}{7}$ and $\frac{7}{5}$ are called **reciprocals**, or numbers that when multiplied equal 1.

$$5 \cdot \frac{1}{5} = 1$$
$$\frac{1}{7} \cdot 7 = 1$$
$$\frac{5}{7} \cdot \frac{7}{5} = 1$$

We can also say that 5 is a reciprocal of $\frac{1}{5}$ and $\frac{1}{5}$ is a reciprocal of 5. Ask students to name the reciprocal of a few other examples, such as 9, $\frac{3}{4}$, b, and $\frac{a}{b}$

If time permits, ask students to use the generalized method to divide other fractions, such as $18 \div \frac{9}{7}$ or $\frac{15}{14} \div \frac{5}{2}$.

11.3 Dividing with or without an Algorithm

15 min

Activity Narrative

This activity serves two goals. It gives students opportunities to use the algorithm developed earlier to divide a wider variety of fractions. It also prompts them to consider the efficiency of different ways of finding quotients.

Students can use any method of reasoning and are not expected to use the algorithm. The numbers in the expressions are selected such that certain methods may be more practical than others. As students work through different problems, they notice that sometimes it is less convenient to use diagrams or other concrete strategies and more efficient to use the algorithm. Other times, it is easier to divide without using the algorithm. Students can choose an approach strategically by looking for and making use of structure in the dividends and divisors (MP7).

Monitor for students who use different ways to find quotients. Here are some likely approaches:

- Reasoning visually about equal groups, using a diagram.
- Reasoning numerically about equal groups, without using a diagram, such as by thinking “How many groups of this fraction is in that fraction?”
- Reasoning in terms of multiplication, such as by writing a corresponding multiplication equation for the division

expression, or by thinking, “What number times the divisor gives the dividend?”

- Applying a generalization from an earlier course, such as by rewriting division by a whole number as multiplication by the reciprocal of that number.
- Applying the fraction division algorithm.

Standards

Addressing **6.NS.A.1**

Instructional Routines

- 5 Practices

Launch

Keep students in groups of 2. Ask each partner to choose at least four quotients to calculate, making sure that all quotients are found by the group. Give students 5 minutes of quiet work time, followed by 2–3 minutes to discuss their responses with a partner.

Select students with different strategies, such as those described in the Activity Narrative, and ask them to share later. It is not essential to discuss every strategy listed, but try to highlight 2–3 different approaches. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven’t shared recently.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide blank tape diagrams for students to show their reasoning. *Supports accessibility for: Organization, Attention*

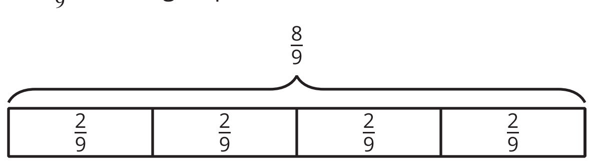
Student Task Statement

Calculate at least four quotients. Show your reasoning.

1. $\frac{8}{9} \div 4$
2. $\frac{9}{12} \div \frac{6}{12}$
3. $3\frac{1}{3} \div \frac{2}{9}$
4. $\frac{9}{2} \div \frac{3}{8}$
5. $1\frac{2}{5} \div 3$
6. $6\frac{1}{4} \div \frac{10}{3}$

Student Response

1. $\frac{2}{9}$ (or equivalent). Sample reasoning:
   - $\frac{8}{9}$ put into 4 equal groups means $\frac{2}{9}$ in each group.



   - Dividing by 4 is the same as multiplying by $\frac{1}{4}$, and $\frac{8}{9} \cdot \frac{1}{4} = \frac{8}{36}$, which is $\frac{2}{9}$.

2. $\frac{3}{2}$ or $1\frac{1}{2}$ (or equivalent). Sample reasoning:
    - There is 1 full group of $\frac{6}{12}$ in $\frac{9}{12}$ plus another $\frac{3}{12}$, which is $\frac{1}{2}$ a group of $\frac{6}{12}$.
    - $\frac{9}{12}$ and $\frac{6}{12}$ are equivalent to $\frac{3}{4}$ and $\frac{2}{4}$. There are $1\frac{1}{2}$ groups of $\frac{2}{4}$ in $\frac{3}{4}$.
3. 15. Sample reasoning:
    - $3\frac{1}{3}$ is $\frac{10}{3}$, which is $\frac{30}{9}$. There are 15 groups of $\frac{2}{9}$ in $\frac{30}{9}$.
    - $\frac{10}{3} \cdot \frac{9}{2} = \frac{90}{6}$, which is 15.
4. 12. Sample reasoning:
    - $\frac{9}{2}$ is equivalent to $\frac{36}{8}$. There are 12 groups $\frac{3}{8}$s in $\frac{36}{8}$.
    - $\frac{9}{2} \cdot \frac{8}{3} = \frac{72}{6} = 12$
5. $\frac{7}{15}$. Sample reasoning:
    - $\frac{7}{5} \div 3 = \frac{7}{5} \cdot \frac{1}{3}$, which is $\frac{7}{15}$.
6. $\frac{15}{8}$ or $1\frac{7}{8}$. Sample reasoning:
    - $\frac{25}{4} \cdot \frac{3}{10} = \frac{75}{40}$, which is $1\frac{35}{40}$ or $1\frac{7}{8}$.

Building on Student Thinking

Having just generalized $n \div \frac{a}{b}$ as $n \cdot \frac{b}{a}$, students who use the algorithm to find $\frac{8}{9} \div 4$ might identify the reciprocal of $\frac{8}{9}$ and write $4 \cdot \frac{9}{8}$. Urge students to:

- Check whether their multiplication expression corresponds to division expression with $\frac{8}{9}$ as the dividend and 4 as the divisor.
- Verify their answer using another approach and their understanding of division. If they find $4\frac{1}{2}$ to be the quotient, ask if $\frac{8}{9}$ divided by 4 gives $4\frac{1}{2}$, or if $4 \cdot 4\frac{1}{2}$ equals $\frac{8}{9}$.

If needed, remind students that a whole number can be written as a fraction with a denominator of 1, so 4 is equivalent to $\frac{4}{1}$, and its reciprocal is $\frac{1}{4}$.

Are You Ready for More?

Suppose you have a quart of grape juice and a quart of milk. You pour 1 cup of the grape juice into the milk and mix it up. Then you pour 1 cup of this mixture back into the grape juice.

Which liquid is more contaminated? Explain how you know. (Note: 1 quart is equal to 4 cups.)

Extension Student Response

They are equally contaminated. Sample reasoning: Adding 1 cup of grape juice to the quart of milk creates a 5-cup mixture that is 1 part grape juice and 4 parts milk. In 1 cup of this mixture there are $\frac{1}{5}$ cup of juice and $\frac{4}{5}$ cup of milk.

- Removing 1 cup of the mixture from the 5 cups leaves a mixture with $\frac{4}{5}$ cup of juice and $3\frac{1}{5}$ cups of milk.
- Adding 1 cup of this mixture back to the 3 cups of grape juice makes a new mixture with $3\frac{1}{5}$ cups of grape juice and

$\frac{4}{5}$ cup of milk.

The ratio of host liquid to new liquid is the same in both mixtures.

Activity Synthesis

The purpose of this discussion is to highlight that, depending on the numbers involved, some ways to reason about fraction division may be more practical than others. Sometimes, such as when the denominator of the divisor is the same as or is a factor of that in the dividend, reasoning about equal groups—with or without diagrams—may be convenient. Other times, the fractions don't lend themselves to be easily represented with diagrams or seen in terms of equal groups. Using a generalization or an algorithm might be much more productive.

Choose 3–4 problems that can illustrate different methods being favored for different problems. For instance, students may be more likely to opt for the algorithm to calculate the last two quotients than to find $\frac{9}{12} \div \frac{6}{12}$.

Invite previously selected students to share their responses and reasoning. Sequence the discussion of the approaches for each quotient from less common to more common. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- "For which division problems was it quick to draw and use diagrams? For which problems were diagrams not convenient?"
- "For which division problems was the algorithm handy and preferable? For which problems was it less so?"
- "Were there divisions that you were able to do mentally without drawing a diagram or recording calculations on paper? What do you notice about the fractions in those problems?"

Lesson Synthesis

The goal of this discussion is for students to reflect on the different strategies they have learned for finding quotients and consider how the numbers in the division could inform their choice of strategy.

Invite students to name some strategies for dividing two numbers. Then, display a series of division expressions for all to see:

A. $\frac{21}{8} \div \frac{7}{4}$

B. $8 \div \frac{4}{7}$

C. $\frac{5}{3} \div \frac{17}{21}$

D. $\frac{40}{7} \div 8$

E. $6\frac{2}{5} \div 1\frac{3}{5}$

F. $\frac{9}{11} \div \frac{2}{3}$

Give students a minute to examine the expressions. Prompt them to identify a division problem that they would prefer to calculate with an algorithm and one that they could comfortably answer another way. Ask questions such as:

- "Which quotients can you find using an algorithm?" (All of them)
- "Which quotients can you efficiently find without using an algorithm? How?" (Sample responses:
  - For A, we can think of $\frac{7}{4}$ as $\frac{14}{8}$, ask how many $\frac{14}{8}$s are in $\frac{21}{8}$, and reason that it is $\frac{21}{14}$ or $\frac{3}{2}$.

- For B, we can think of 8 as $\frac{56}{7}$ and think about how many $\frac{4}{7}$s are in $\frac{56}{7}$.
- For D, we can think about putting 40 sevenths into 8 groups, which gives 5 sevenths per group.
- For E, we can think in terms of multiplication: $? \cdot \frac{8}{5} = \frac{32}{5}$ and see that the unknown factor is 4.)

- "Which quotients would you prefer to find with an algorithm? Why?" (C and F, as they are not as practical to calculate using other ways)

Emphasize that although we now have a reliable and efficient method to divide any number by any fraction, sometimes it is still easier and more natural to think of the quotient in terms of a multiplication problem with a unknown factor or to reason about equal-size groups.

11.4 Finding Quotients of Fractions

Cool-down

5 min

Standards

Addressing **6.NS.A.1**

Student Task Statement

Calculate each quotient. Show your reasoning.

1. $\frac{24}{25} \div \frac{4}{5}$
2. $4 \div \frac{2}{7}$

Student Response

1. $\frac{6}{5}$ (or equivalent). Sample reasoning:
   - $\frac{4}{5}$ is $\frac{20}{25}$. There is 1 full group of $\frac{20}{25}$ in $\frac{24}{25}$. The leftover $\frac{4}{25}$ is $\frac{1}{5}$ of a group. There is a total of $1\frac{1}{5}$ groups.
   - $\frac{24}{25} \cdot \frac{5}{4} = \frac{120}{100} = \frac{6}{5}$
2. 14 (or equivalent). Sample reasoning:
   - $4 \cdot \frac{7}{2} = \frac{28}{2} = 14$
   - There are 28 one-sevenths in 4 so there are half as many two-sevenths. Half of 28 is 14.

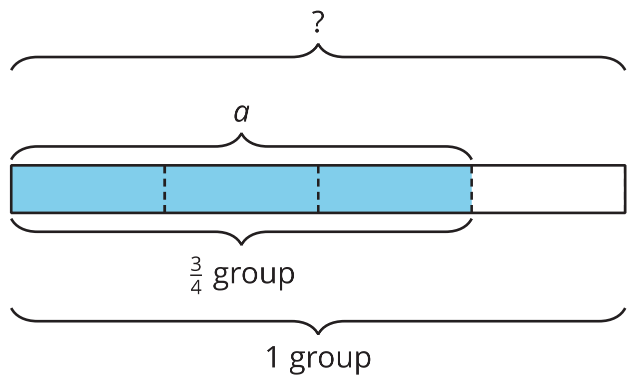
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 11 Summary

We can think of the division $a \div \frac{3}{4} =?$ in terms of finding the size of 1 group: "If there is a in $\frac{3}{4}$ of a group, how much is in 1 group?" or "If $\frac{3}{4}$ of a number is a, what is that number?"

On a tape diagram, we can show $\frac{3}{4}$ of a group having a value of a and the whole group having an unknown value.



If $\frac{3}{4}$ of a number is a, then to find the number, we can first divide a by 3 to find $\frac{1}{4}$ of the number. Then we multiply the result by 4 to find the number.

These steps can be written as: $a \div 3 \cdot 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so we can also write the steps as: $a \cdot \frac{1}{3} \cdot 4$, which is $a \cdot \frac{4}{3}$.

In other words: $a \div \frac{3}{4} = a \cdot \frac{4}{3}$.

In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the **reciprocal** of the fraction. Reciprocals are numbers that when multiplied equal 1.

Glossary

- reciprocal

Lesson 11 Practice Problems

1 Student Task Statement

Select **all** the statements that show correct reasoning for finding $\frac{14}{15} \div \frac{7}{5}$.

A. Multiplying $\frac{14}{15}$ by 5 and then by $\frac{1}{7}$.

B. Dividing $\frac{14}{15}$ by 5, and then multiplying by $\frac{1}{7}$.

C. Multiplying $\frac{14}{15}$ by 7, and then multiplying by $\frac{1}{5}$.

D. Multiplying $\frac{14}{15}$ by 5 and then dividing by 7.

E. Multiplying $\frac{15}{14}$ by 7 and then dividing by 5.

Solution

A, D

2 Student Task Statement

Clare said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$. She reasoned: $\frac{4}{3} \cdot 5 = \frac{20}{3}$, and $\frac{20}{3} \div 2 = \frac{10}{3}$.

Explain why Clare's answer and reasoning are incorrect. Find the correct quotient.

Solution

The correct quotient is $\frac{8}{15}$. Sample reasoning:

- Clare should have multiplied $\frac{4}{3}$ by 2 to find how many groups of $\frac{1}{2}$ are in $\frac{4}{3}$ and then divided the result by 5.
- Clare divided the fraction $\frac{4}{3}$ by the fraction $\frac{2}{5}$ instead of $\frac{5}{2}$.

3 Student Task Statement

Find the value of $\frac{15}{4} \div \frac{5}{8}$. Show your reasoning.

Solution

6. Sample reasoning: There are $\frac{15}{4} \cdot 8$ or 30 groups of $\frac{1}{8}$ in $\frac{15}{4}$. If five $\frac{1}{8}$s make a group, then the number of groups is $\frac{1}{5}$ of 30, which is 6.

4 **Student Task Statement**

Consider the problem: Kiran has $2\frac{3}{4}$ pounds of flour. When he divides the flour into equal-size bags, he fills $4\frac{1}{8}$ bags. How many pounds fit in each bag?

Write a multiplication equation and a division equation to represent the question. Then, find the answer and show your reasoning.

Solution

$\frac{2}{3}$ pound per bag. Sample reasoning: $4\frac{1}{8} \cdot ? = 2\frac{3}{4}$ can be written as $2\frac{3}{4} \div 4\frac{1}{8} = ?$. Using the algorithm to divide: $2\frac{3}{4} \div 4\frac{1}{8} = \frac{11}{4} \div \frac{33}{8} = \frac{11}{4} \cdot \frac{8}{33} = \frac{2}{3}$.

5 from Unit 4, Lesson 10

Student Task Statement

Divide $4\frac{1}{2}$ by each of these unit fractions.

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{1}{6}$

Solution

a. 36

b. 18

c. 27

6 from Unit 4, Lesson 9

Student Task Statement

A phone's battery is out of power. After charging for $\frac{1}{3}$ of an hour, the battery is at $\frac{2}{5}$ of its full power. How long (in total) will it take the battery to charge completely?

Decide whether each equation can represent the situation.

a. $\frac{1}{3} \cdot ? = \frac{2}{5}$

b. $\frac{1}{3} \div \frac{2}{5} = ?$

c. $\frac{2}{5} \div \frac{1}{3} = ?$

d. $\frac{2}{5} \cdot ? = \frac{1}{3}$

Solution

a. No

b. Yes

c. No

d. Yes

7 from Unit 4, Lesson 8

Student Task Statement

Elena is filling a bucket with water. When her bucket is $\frac{2}{5}$ full, the water weighs $2\frac{1}{2}$ pounds. How much would the water weigh if her bucket is full?

a. Write multiplication and division equations to represent the question.

b. Draw a diagram to show the relationship between the quantities and to find the answer.

Solution

a. $\frac{2}{5} \cdot ? = 2\frac{1}{2}$ (or equivalent), $2\frac{1}{2} \div \frac{2}{5} = ?$

b. $6\frac{1}{4}$ pounds. Sample reasoning: