Solving More Systems

Let's solve systems of equations.

14.1 Math Talk: Solving Systems

Solve each system mentally.

- $\begin{cases} x = 8 \\ y = \text{-}11 \end{cases}$

- $\begin{cases} x = 5 \\ y = x - 7 \end{cases}$

- $\begin{cases} y = 3x - 2 \\ y = 4 \end{cases}$

- $\begin{cases} y = 2x + 3 \\ y = \frac{1}{2}(4x + 3) \end{cases}$

14.2 Challenge Yourself

Here are a lot of systems of equations:

A $\begin{cases} y = 4 \\ x = -5y + 6 \end{cases}$

B $\begin{cases} y = 7 \\ x = 3y - 4 \end{cases}$

C $\begin{cases} y = \frac{3}{2}x + 7 \\ x = -4 \end{cases}$

D $\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$

E $\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$

F $\begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$

G $\begin{cases} y = 3x \\ x = -2y + 56 \end{cases}$

H $\begin{cases} x = 2y - 15 \\ y = -2x \end{cases}$

I $\begin{cases} 3x + 4y = 10 \\ x = 2y \end{cases}$

J $\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$

K $\begin{cases} y = -2x + 5 \\ 2x + 3y = 31 \end{cases}$

L $\begin{cases} x + y = 10 \\ x = 2y + 1 \end{cases}$

1. Without solving, identify 3 systems that you think would be the least difficult to solve and 3 systems that you think would be the most difficult to solve. Be prepared to explain your reasoning.

2. Choose 4 systems to solve. At least one should be from your "least difficult" list, and one should be from your "most difficult" list.

14.3 Five Does Not Equal Seven

Tyler looks at this system of equations:

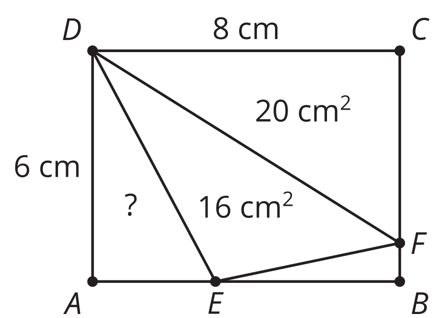
$$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$$

He says, "Just looking at the system, I can see it has no solution. If you add 2 numbers, that sum can't be equal to 2 different numbers."

Do you agree with Tyler?

Are you ready for more?

In rectangle $ABCD$, side AB is 8 centimeters and side BC is 6 centimeters. F is a point on BC and E is a point on AB. The area of triangle DFC is 20 square centimeters, and the area of triangle DEF is 16 square centimeters. What is the area of triangle AED?



Lesson 14 Summary

When we have a system of linear equations where one of the equations is of the form $y = [\text{stuff}]$ or $x = [\text{stuff}]$, we can solve it algebraically by using a technique called *substitution*. The basic idea is to replace a variable with an expression that it is equal to (so the expression is like a substitute for the variable). For example, let's start with the system:

$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

Because we know that $y = 5x$, we can substitute $5x$ for y in the equation $2x - y = 9$,

$$2x - (5x) = 9$$

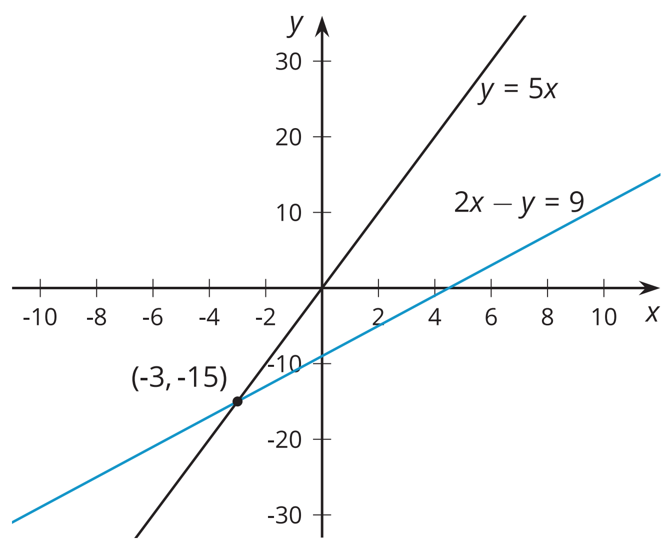
and then solve the equation for x,

$$x = \text{-}3$$

We can find y using either equation. Using the first one, $y = 5 \cdot \text{-}3$.
So $(\text{-}3, \text{-}15)$ is the solution to this system.

We can verify this by looking at the graphs of the equations in the system:

Sure enough! They intersect at $(\text{-}3, \text{-}15)$.



We didn't know it at the time, but we were actually using substitution in the last lesson as well. In that lesson, we looked at the system

$$\begin{cases} y = 2x + 6 \\ y = \text{-}3x - 4 \end{cases}$$

We substituted $2x + 6$ for y into the second equation to get $2x + 6 = \text{-}3x - 4$. Go back and check for yourself!