

# Polynomial Division (Part 2)

Let's learn a different way to divide polynomials.

## 13.1 Notice and Wonder: Different Divisions

What do you notice? What do you wonder?

$$\begin{array}{r} 2 \\ 11 \overline{) 2772} \\ \underline{22} \phantom{00} \\ 5 \phantom{00} \end{array}$$

$$\begin{array}{r} 25 \\ 11 \overline{) 2772} \\ \underline{22} \phantom{00} \\ 57 \\ \underline{55} \phantom{00} \\ 2 \phantom{00} \end{array}$$

$$\begin{array}{r} 252 \\ 11 \overline{) 2772} \\ \underline{22} \phantom{00} \\ 57 \\ \underline{55} \phantom{00} \\ 22 \\ \underline{22} \phantom{00} \\ 0 \end{array}$$

$$\begin{array}{r} 2x^2 \\ x+1 \overline{) 2x^3 + 7x^2 + 7x + 2} \\ \underline{-2x^3 - 2x^2} \phantom{00} \\ 5x^2 + 7x \phantom{00} \end{array}$$

## 13.2 Polynomial Long Division

1. Diego used the long division shown here to figure out that  $6x^2 - 7x - 5 = (2x + 1)(3x - 5)$ . Show what it would look like if he had used a diagram.

$$\begin{array}{r} 3x - 5 \\ 2x + 1 \overline{) 6x^2 - 7x - 5} \\ \underline{-6x^2 - 3x} \phantom{00} \\ -10x - 5 \\ \underline{10x + 5} \\ 0 \end{array}$$

|    |                 |  |
|----|-----------------|--|
|    |                 |  |
| 2x | 6x <sup>2</sup> |  |
| 1  |                 |  |

Pause here for a whole-class discussion.

2.  $(x - 2)$  is a factor of  $2x^3 - 7x^2 + x + 10$ , which means there is some other factor  $A$  where  $2x^3 - 7x^2 + x + 10 = (x - 2)(A)$ . Finish the division started here to find the value of  $A$ .

$$\begin{array}{r} 2x^2 \\ x - 2 \overline{) 2x^3 - 7x^2 + x + 10} \\ \underline{-2x^3 + 4x^2} \end{array}$$

3. Jada used the diagram shown here to figure out that  $2x^3 + 13x^2 + 16x + 5 = (2x + 1)(x^2 + 6x + 5)$ . Show what it would look like if she had used long division.

|      |        |         |       |
|------|--------|---------|-------|
|      | $x^2$  | $6x$    | $5$   |
| $2x$ | $2x^3$ | $12x^2$ | $10x$ |
| $1$  | $x^2$  | $6x$    | $5$   |

$$2x + 1 \overline{) 2x^3 + 13x^2 + 16x + 5}$$

 **Are you ready for more?**

1. What is  $(x^4 - 1) \div (x - 1)$ ?

2. Use your response to predict what  $(x^7 - 1) \div (x - 1)$  is, and then use division to check your prediction.

## 13.3 More Long Division

Here are some polynomial functions with known factors. Rewrite each polynomial as a product of linear factors using long division.

1.  $A(x) = x^3 - 7x^2 - 16x + 112, (x - 7)$

$$\begin{array}{r} x^2 \\ x - 7 \overline{) x^3 - 7x^2 - 16x + 112} \\ \underline{-x^3 + 7x^2} \phantom{- 16x + 112} \end{array}$$

2.  $C(x) = x^3 - 3x^2 - 13x + 15, (x + 3)$



## 13.4 Missing Numbers

Here are pairs of equivalent expressions, one in standard form and the other in factored form. Find the missing numbers.

1.  $x^2 + 9x + 14$  and  $(x + 2)(x + \boxed{\phantom{00}})$

2.  $x^2 - 9x + 20$  and  $(x - \boxed{\phantom{00}})(x - \boxed{\phantom{00}})$

3.  $2x^2 + 2x - 24$  and  $2(x + \boxed{\phantom{00}})(x - 3)$

4.  $\boxed{\phantom{00}}x^3 + 11x^2 - 17x + 6$  and  $(-x + 3)(2x - 1)(x - 2)$

5.  $6x^3 + 2x^2 - 16x + 8$  and  $(x - 1)(2x + 4)(\boxed{\phantom{00}}x - 2)$

6.  $2x^3 + 7x^2 - 7x - 12$  and  $(2x - 3)(x + \boxed{\phantom{00}})(x + \boxed{\phantom{00}})$

7.  $x^3 + 6x^2 + \boxed{\phantom{00}}x - 10$  and  $(x + 2)(x - 1)(x + \boxed{\phantom{00}})$



## Lesson 13 Summary

In earlier grades, we learned that one way to divide numbers, like 1,573 divided by 11, is by using long division.

$$\begin{array}{r} 1 \\ 11 \overline{) 1573} \\ \underline{11} \phantom{00} \\ 4 \phantom{00} \end{array}$$

$$\begin{array}{r} 14 \\ 11 \overline{) 1573} \\ \underline{11} \phantom{00} \\ 47 \phantom{00} \\ \underline{44} \phantom{00} \\ 3 \phantom{00} \end{array}$$

$$\begin{array}{r} 143 \\ 11 \overline{) 1573} \\ \underline{11} \phantom{00} \\ 47 \phantom{00} \\ \underline{44} \phantom{00} \\ 33 \phantom{00} \\ \underline{33} \phantom{00} \\ 0 \end{array}$$

Here the division has been completed in stages, focusing on the highest power of 10 (1,000) in the dividend 1,573, and working down. This long division shows that  $1,573 = (11)(143)$ .

Similar to integers, we can also use long division on polynomials. Instead of focusing on powers of 10, in polynomial long division we focus on powers of  $x$ . Just as we started with the highest power or 10, we start with the highest power of  $x$ , the leading term, and work down to the constant term. For example, here is  $x^3 + 5x^2 + 7x + 3$  divided by  $x + 1$  completed in three stages. Notice how terms of the same degree are in the same columns.

$$\begin{array}{r} x^2 \\ x + 1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \phantom{00} \\ 4x^2 + 7x \phantom{00} \end{array}$$

$$\begin{array}{r} x^2 + 4x \\ x + 1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \phantom{00} \\ 4x^2 + 7x \phantom{00} \\ \underline{-4x^2 - 4x} \phantom{00} \end{array}$$

$$\begin{array}{r} x^2 + 4x + 3 \\ x + 1 \overline{) x^3 + 5x^2 + 7x + 3} \\ \underline{-x^3 - x^2} \phantom{00} \\ 4x^2 + 7x \phantom{00} \\ \underline{-4x^2 - 4x} \phantom{00} \\ 3x + 3 \end{array}$$

At each stage, the focus is only on the term with the largest exponent that's left. At the conclusion, we can see that  $x^3 + 5x^2 + 7x + 3 = (x + 1)(x^2 + 4x + 3)$ .