Unit 2, Lesson 7

Reasoning about Similarity with Transformations

Let’s describe similar triangles.

7.1 AAA Triangles

1. Sketch 2 triangles with all pairs of corresponding angles congruent, and with all pairs of corresponding side lengths in the same proportion.

2. Label your triangles ABC and DEF so that angle A is congruent to angle D, angle B is congruent to angle E, and angle C is congruent to angle F. Label each side with its length.
3. Trade your pair of triangles with your partner, then check that the corresponding angles are congruent and that the sides are labeled with the correct measurements.
4. Suggest to your partner any ways in which they might improve their sketch to be more accurate, and then trade your triangles back.

7.2 Stretched or Distorted? Triangles

1. Must the two triangles you drew be similar by the definition? Explain your reasoning.

2. Switch sketches with your partner. Find a sequence of rigid motions and dilations that will take one of their triangles onto the other.

3. Will the same sequence work for your triangles?

Are you ready for more?

How many sequences of transformations are there that take one similar triangle to the other? Explain or show your reasoning.

7.3 Invisible Triangles: Similarity

Player 1: You are the transformer. Take the transformer card.

Player 2: Select a triangle card. Do not show it to anyone. Study the diagram to figure out which sides and which angles correspond. Tell Player 1 what you have figured out about which sides and angles correspond.

Player 1: Take notes about what your partner tells you so that you know which parts of the triangles correspond. Think of a sequence of rigid motions and dilations you could tell your partner to get them to take one of their triangles onto the other. Be specific in your language. The notes on your card can help with this.

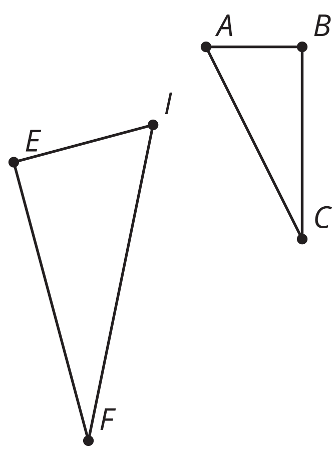
Player 2: Listen to the instructions from the transformer. Use tracing paper to follow the instructions. Draw the image after each step. Let the transformer know when they have lined up 1, 2, or all 3 pairs of vertices on your triangles.

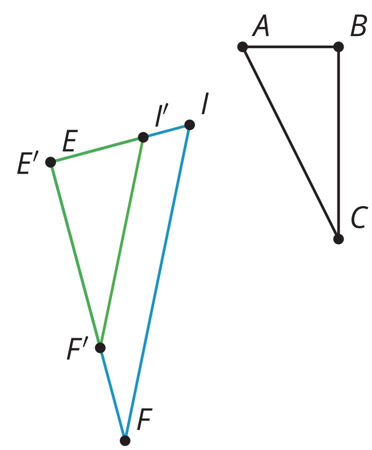
For the next round, change roles so that each person has a chance to be the transformer. Draw a new triangle card and repeat the steps of the game.

Lesson 7 Summary

One figure is similar to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second. By the properties of dilations and rigid motions, similar figures have corresponding angles that are congruent and pairs of corresponding side lengths that are in the same proportion.

In the case of triangles, the converse of this statement is true as well. If a pair of triangles has all pairs of corresponding side lengths in the same proportion, and all pairs of corresponding angles are congruent, then the triangles must be similar. Imagine any pair of triangles in which all pairs of corresponding side lengths are in the same proportion and all pairs of corresponding angles are congruent. The same sequence of rigid motions and dilations will work to show that the triangles are similar.





For example, triangle EFI was dilated, using E as the center, by the scale factor given by $\frac{BC}{EF}$. Because we wisely chose the scale factor this way, we know that side BC is congruent to side $E'F'$. We already know that all pairs of corresponding angles are congruent, which means we have enough information to use the Angle-Side-Angle Triangle Congruence Theorem to prove that triangle $E'F'I'$ is congruent to triangle ABC.

That means that triangle ABC can be lined up exactly with a dilation of triangle EFI, which is the definition of similarity. It doesn't matter what the triangles look like or where we start. We can always define a dilation that made one pair of corresponding sides congruent, and then use the Angle-Side-Angle Triangle Congruence Theorem to finish proving that there is a sequence of dilations and rigid motions that takes one triangle onto the other.