Rewriting Quadratic Expressions in Factored Form (Part 2)

Let's write some more expressions in factored form.

7.1 Sums and Products

1. The product of the integers 2 and -6 is -12. List all the other pairs of integers whose product is -12.

2. Of the pairs of factors you found, list all pairs that have a positive sum. Explain why they all have a positive sum.

3. Of the pairs of factors you found, list all pairs that have a negative sum. Explain why they all have a negative sum.

7.2 Negative Constant Terms

1. Each row of this table should have a pair of equivalent expressions. Complete the table. If you get stuck, consider drawing a diagram.

factored form	standard form
$(x+5)(x+6)$	
	$x^2+13x+30$
$(x-3)(x-6)$	
	$x^2-11x+18$

2. Each row in this table should have a pair of equivalent expressions. Complete the table. If you get stuck, consider drawing a diagram.

factored form	standard form
$(x+12)(x-3)$	
	$x^2-9x-36$
	$x^2-35x-36$
	$x^2+35x-36$

3. How are the two tables similar and different?

7.3 Factors of 100 and -100

1. Consider the expression $x^2 + bx + 100$.

   Complete the first table with all factor pairs of 100 that would give positive values of b, and the second table with factors that would give negative values of b.

   For each pair, state the b value they produce. (Use as many rows as needed.)

   positive value of b

factor 1	factor 2	b (positive)

   negative value of b

factor 1	factor 2	b (negative)

2. Consider the expression $x^2 + bx - 100$.

   Complete the first table with all factor pairs of -100 that would result in positive values of b, the second table with factors that would result in negative values of b, and the third table with factors that would result in a zero value of b.

   For each pair of factors, state the b value they produce. (Use as many rows as there are pairs of factors. You may not need all the rows.)

   positive value of b

factor 1	factor 2	b (positive)

   negative value of b

factor 1	factor 2	b (negative)

zero value of b

factor 1	factor 2	b (zero)

3. Write each expression in factored form:

   a. $x^2 - 25x + 100$

   b. $x^2 + 15x - 100$

   c. $x^2 - 15x - 100$

   d. $x^2 + 99x - 100$

Are you ready for more?

How many different integers b can you find so that the expression $x^2 + 10x + b$ can be written in factored form?

Lesson 7 Summary

When we rewrite expressions in factored form, it is helpful to remember that:

- Multiplying two positive numbers or two negative numbers results in a positive product.
- Multiplying a positive number and a negative number results in a negative product.

This means that if we want to find two factors whose product is 10, the factors must both be positive or both be negative. If we want to find two factors whose product is -10, one of the factors must be positive and the other negative.

Suppose we wanted to rewrite $x^2 - 8x + 7$ in factored form. Recall that subtracting a number can be thought of as adding the opposite of that number, so that expression can also be written as $x^2 + \text{-}8x + 7$. We are looking for two numbers that:

- Have a product of 7. The candidates are 7 and 1, and -7 and -1.
- Have a sum of -8. Only -7 and -1 from the list of candidates meet this condition.

The factored form of $x^2 - 8x + 7$ is therefore $(x + \text{-}7)(x + \text{-}1)$ or, written another way, $(x - 7)(x - 1)$.

To write $x^2 + 6x - 7$ in factored form, we would need two numbers that:

- Multiply to make -7. The candidates are 7 and -1, and -7 and 1.
- Add up to 6. Only 7 and -1 from the list of candidates add up to 6.

The factored form of $x^2 + 6x - 7$ is $(x + 7)(x - 1)$.