Linear Functions and Models

Let's investigate linear functions.

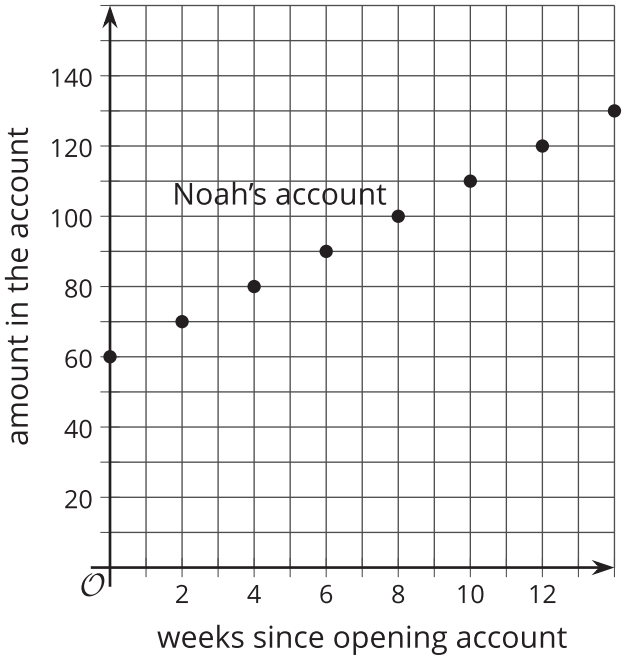
7.1 Which Is Growing Faster?

Noah is depositing money in his account every week to save money. The graph shows the amount he has saved as a function of time since he opened his account.

Elena opened an account the same day as Noah. The amount of money E in her account is given by the function $E = 8w + 60$, where w is the number of weeks since the account was opened.

1. Who started out with more money in their account? Explain how you know.

2. Who is saving money at a faster rate? Explain how you know.



7.2 Is It Filling Up or Draining Out?

There are four tanks of water.

- The amount of water in gallons, A, in Tank A is a function of time in minutes, t, and can be represented by the equation $A = 200 + 8t$.
- The amount of water in gallons, B, in Tank B is a function of time in minutes, t. The amount of water starts at 400 gallons and is decreasing by 5 gallons per minute.

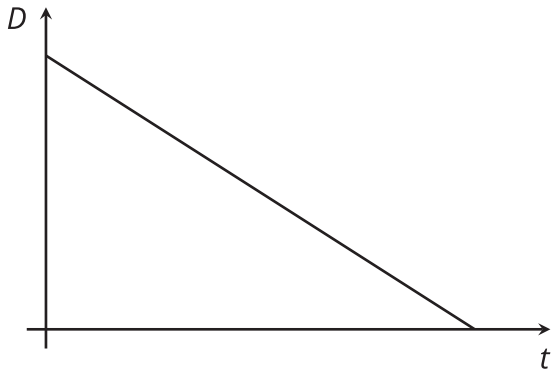
1. Which tank started out with more water?

2. Write an equation representing the relationship between B and t.

3. One tank is filling up. The other is draining out. Which is which? How can you tell?

4. The amount of water in gallons, C, in Tank C is a function of time in minutes, t, and can be represented by the equation $C = 800 - 7t$. Is it filling up or draining out? Can you tell just by looking at the equation?

5. The graph of the function for the amount of water in gallons, D, in Tank D at time t is shown. Is it filling up or draining out? How do you know?



Are you ready for more?

1. Pick a tank that is draining out. How long does it take for that tank to drain? What percent of the tank is full when 30% of that time has elapsed? When 70% of the time has elapsed?

2. What point in the plane is 30% of the way from $(0, 15)$ to $(5, 0)$? 70% of the way?

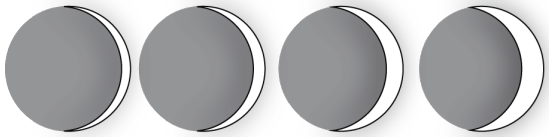
3. What point in the plane is 30% of the way from $(3, 5)$ to $(8, 6)$? 70% of the way?

7.3 A Candle and the Moon

1. A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.
   a. When do you think the candle will burn out completely?

   b. Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

2. On the first day after the new moon, 2% of the moon's surface that we can see is illuminated. On the second day, 6% is illuminated.



Use this information to predict the days on which the moon's surface that we can see is 50% illuminated and 100% illuminated.

7.4 Shadows

When the sun was directly overhead, the stick had no shadow. After 20 minutes, the shadow was 10.5 centimeters long. After 60 minutes, it was 26 centimeters long.

1. Use this information to estimate how long it will be after 95 minutes.

2. After 95 minutes, the shadow measured 38.5 centimeters. How does this compare to your estimate?

3. Is the length of the shadow a function of time? If so, is it linear? Explain your reasoning.

Lesson 7 Summary

Suppose a car is traveling at 30 miles per hour. The relationship between the time in hours and the distance in miles is a proportional relationship.

We can represent this relationship with an equation of the form $d = 30t$, where distance is a function of time (since each input of time has exactly one output of distance).

Or we could write the equation $t = \frac{1}{30}d$ instead, where time is a function of distance (since each input of distance has exactly one output of time).

These equations are examples of a mathematical model. A mathematical model is a mathematical object, like an equation, a function, or a geometric figure, that we use to represent a real-life situation. Sometimes a situation can be modeled by a linear function. We have to analyze the information we are given and use judgment about whether using a linear model is a reasonable thing to do. We must also be aware that the model may make imprecise predictions or may only be appropriate for certain ranges of values.

More generally, if we represent a linear function with an equation like $y = mx + b$, then b is the initial value (which is 0 for proportional relationships), and m is the rate of change of the function.

If m is positive, the function is increasing.

If m is negative, the function is decreasing.

If we represent a linear function in a different way, say with a graph, we can use what we know about graphs of lines to find the and values and, if needed, write an equation.