Methods for Multiplying Decimals

Let's look at some ways we can represent multiplication of decimals.

16.1 Which Three Go Together: Multiplication Expressions

Which three go together? Why do they go together?

A

$$(0.1) \cdot 2 \cdot 3$$

B

$$3 \cdot (0.2)$$

C

$$(0.1) \cdot 3$$

D

$$6 \cdot \frac{1}{10}$$

16.2 Using Properties of Numbers to Reason about Multiplication

Elena and Noah used different methods to compute $(2.4) \cdot (1.3)$. Both calculations were correct.

Elena's Method	Noah's Method
$(2.4) \cdot 10 = 24$	$2.4 = \frac{24}{10}$
$(1.3) \cdot 10 = 13$	$1.3 = \frac{13}{10}$
$24 \cdot 13 = 312$	$\frac{24}{10} \cdot \frac{13}{10} = \frac{312}{100}$
$312 \div 100 = 3.12$	$\frac{312}{100} = 3.12$

1. Analyze the two methods, then discuss these questions with your partner.
    - Which method makes more sense to you? Why?
    - What might Elena do to compute $(0.16) \cdot (0.03)$?
    - What might Noah do to compute $(0.16) \cdot (0.03)$?
    - Will the two methods result in the same value?

2. Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain or show your reasoning.

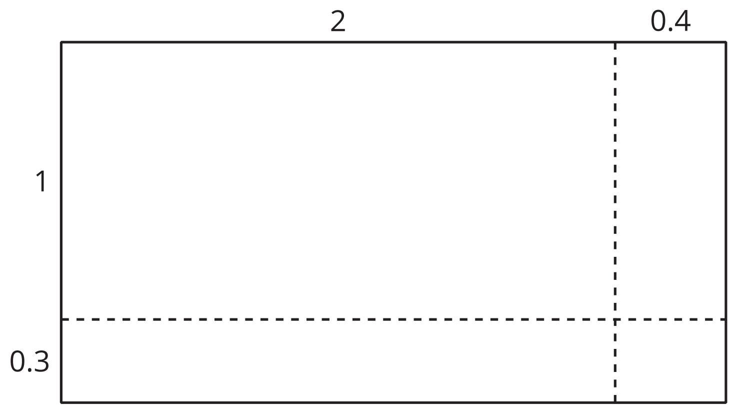
    a. $(2.1) \cdot (4.7)$

    b. $21 \cdot (0.047)$

    c. $(0.021) \cdot (4.7)$

16.3 Connecting Area Diagrams to Calculations with Decimals

1. Here is an area diagram that represents $(2.4) \cdot (1.3)$.



   a. Find the region that represents $(0.4) \cdot (0.3)$. Label it with its area of 0.12.

   b. Label the other regions with their areas.

   c. Find the value of $(2.4) \cdot (1.3)$. Show your reasoning.

2. Here are two ways of calculating $(2.4) \cdot (1.3)$.

$$\begin{array}{rr} & 2.4 \\ \times & 1.3 \\ \hline & 0.12 \\ & 0.6 \\ & 0.4 \\ + & 2 \\ \hline & 3.12 \end{array} \quad \text{partial products}$$

Calculation A

$$\begin{array}{rr} & 2.4 \\ \times & 1.3 \\ \hline & 0.72 \\ + & 2.4 \\ \hline & 3.12 \end{array}$$

Calculation B

   Analyze the calculations and discuss these questions with a partner:

   a. In Calculation A, where do the 0.12 and other partial products come from?

   b. In Calculation B, where do the 0.72 and 2.4 come from?

   c. In each calculation, why are the numbers below the horizontal line aligned vertically the way they are?

3. Find the value of $(3.1) \cdot (1.5)$ in two ways:

   a. Draw and label a diagram. Show your reasoning.

   b. Calculate numerically, without using a diagram. Be prepared to explain your reasoning.

Are you ready for more?

Zhang (JAHNG), or “Chinese yard,” and *li* (LEE), or “Chinese mile,” are two units of length used in China.

1. If 1 li is equal to 150 zhangs, and 1 zhang is approximately 3.645 yards (as used in the United States), about how many yards are in 1 li?

2. There are 1,760 yards in 1 mile (as used in the United States). Estimate how many lis are in 1 mile. Explain your reasoning.

Lesson 16 Summary

Here are three other ways to calculate a product of two decimals, such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

  $$(0.04) \cdot 100 = 4$$
  $$(0.07) \cdot 100 = 7$$

  Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

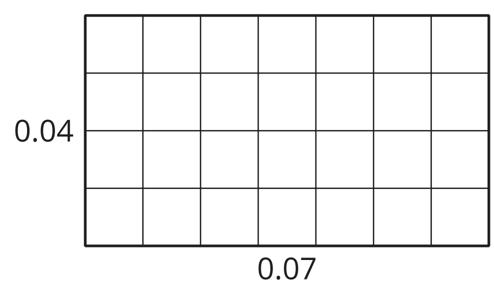
  $$4 \cdot 7 = 28$$
  $$28 \div 10{,}000 = 0.0028$$

- Second, we can write each decimal as a fraction and multiply them.

  $$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10{,}000} = 0.0028$$

- Third, we can use an area diagram. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.

  In this diagram, each small square is 0.01 unit by 0.01 unit. The area of each square, in square units, is therefore $\left(\frac{1}{100} \cdot \frac{1}{100}\right)$, which is $\frac{1}{10{,}000}$.



  Because the rectangle is composed of 28 small squares, the area of the rectangle, in square units, must be:

  $$28 \cdot \frac{1}{10,000} = \frac{28}{10,000} = 0.0028$$

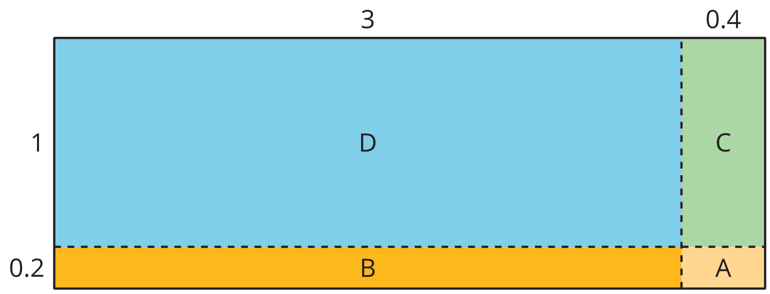
All three calculations show that $(0.04) \cdot (0.07) = 0.0028$.

To find the product of two two-digit numbers, such as $(3.4) \cdot (1.2)$, we can think of finding the area of a rectangle with those numbers, 3.4 units and 1.2 units, as side lengths.

First, we draw a rectangle and partition each side length by place value, into ones and tenths:

$$3.4 = 3 + 0.4$$
$$1.2 = 1 + 0.2$$

Then, we decompose the rectangle into four smaller sub-rectangles and find their areas.



A: $(0.4) \cdot (0.2) = 0.08$

B: $3 \cdot (0.2) = 0.6$

C: $(0.4) \cdot 1 = 0.4$

D: $3 \cdot 1 = 3$

$0.08 + 0.6 + 0.4 + 3 = 4.08$

Each multiplication gives a *partial product* that represents the area of a sub-rectangle. The sum of the four partial products gives the area of the entire rectangle, 4.08 square units.

We can show the same partial-product calculations vertically. Here are two ways:

$$\begin{array}{rr l}
 & 3.4 & \\
\times & 1.2 & \\
\hline
 & {}^{1}\ \ \ \ & \\
 & 0.08 & \text{A} \\
 & 0.6\ \ & \text{B} \\
 & 0.4\ \ & \text{C} \\
+ & 3\ \ \ \ \ & \text{D} \\
\hline
 & 4.08 &
\end{array}
\qquad\qquad
\begin{array}{rr l}
 & 3.4 & \\
\times & 1.2 & \\
\hline
 & {}^{1}\ \ \ \ & \\
 & 0.68 & \text{A} + \text{B} \\
+ & 3.4\ \ & \text{C} + \text{D} \\
\hline
 & 4.08 &
\end{array}$$

The calculation on the left shows four partial products, one for the area of each sub-rectangle.

The calculation on the right shows two partial products:

- 0.68 is the value of $(3.4) \cdot (0.2)$, or the combined area of A and B.
- 3.4 is the value of $(3.4) \cdot 1$, or the combined area of C and D.

In both calculations, adding the partial products gives a total of 4.08, which is the product of $(3.4) \cdot (1.2)$ and the area (in square units) of the entire rectangle.

