Unit 5, Lesson 7

Using Diagrams to Represent Multiplication

Let’s use area diagrams to find products.

7.1 Estimate the Product

For each multiplication expression, choose the best estimate of its value. Be prepared to explain your reasoning.

1. $(6.8) \cdot (2.3)$
    - ○ 1.40
    - ○ 14
    - ○ 140
2. $74 \cdot (8.1)$
    - ○ 5.6
    - ○ 56
    - ○ 560
3. $166 \cdot (0.09)$
    - ○ 1.66
    - ○ 16.6
    - ○ 166
4. $(3.4) \cdot (1.9)$
    - ○ 6.5
    - ○ 65
    - ○ 650

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7.2 Connecting Area Diagrams to Calculations with Whole Numbers

1. Here are three ways of finding the area of a rectangle that is 24 units by 13 units.

Diagram 1

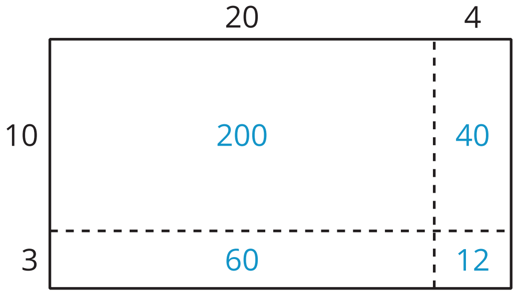


Diagram 2

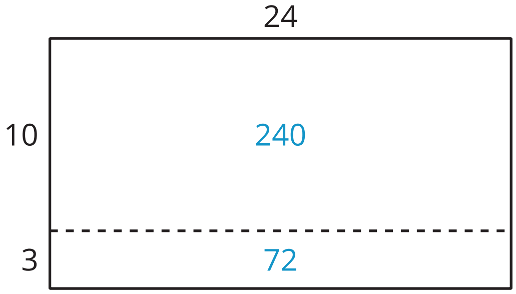
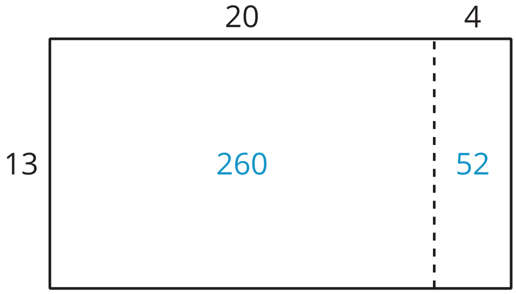


Diagram 3



Discuss with your partner:

a. How are the diagrams the same?

b. How are the diagrams different?

c. If you were to find the area of a rectangle that is 37 units by 19 units, which of the three ways of decomposing the rectangle would you use? Why?

2. Here are two ways to calculate 24 times 13.

$$\begin{array}{r} 2\,4 \\ \times \qquad 1\,3 \\ \hline \begin{array}{r} 1\,2 \\ 6\,0 \\ 4\,0 \\ + \quad 2\,0\,0 \end{array} \\ \hline 3\,1\,2 \end{array} \left.\vphantom{\begin{array}{r} 1 \\ 6 \\ 4 \\ 2 \end{array}}\right\} \text{partial products}$$

Calculation A

$$\begin{array}{r} 2\,4 \\ \times \qquad 1\,3 \\ \hline 7\,2 \\ + \quad 2\,4\,0 \\ \hline 3\,1\,2 \end{array}$$

Calculation B

Discuss with your partner:

a. In Calculation A, where does each partial product—the 12, 60, 40, and 200—come from?

b. In Calculation B, where do 72 and 240 come from?

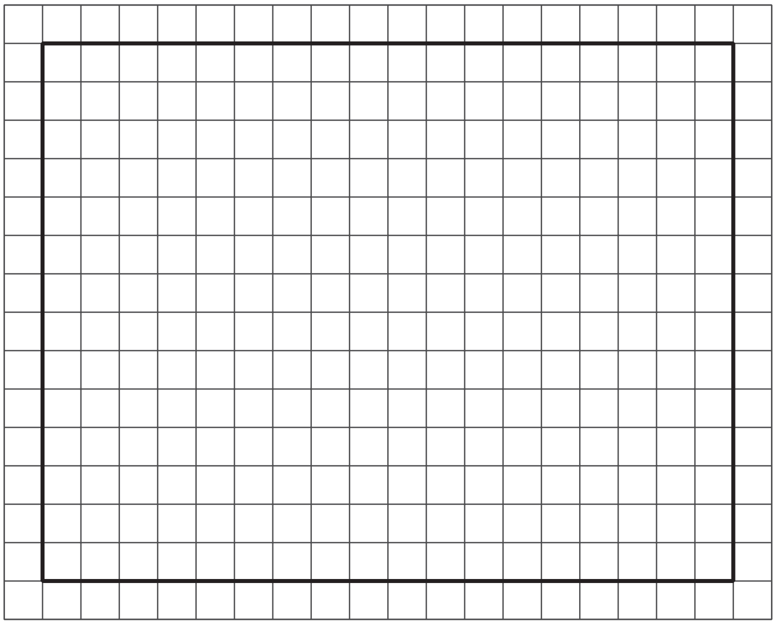
c. Which diagram in the first question corresponds to Calculation A? Which one corresponds to Calculation B? How do you know?

3. Find the product of 18 and 14 in two ways:

a. Calculate numerically.

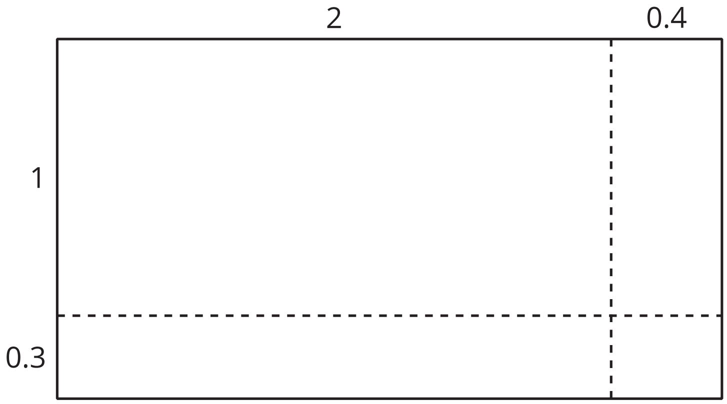
$$\begin{array}{r} 1\,8 \\ \times \qquad 1\,4 \\ \hline \end{array}$$

b. Find the area, in square units, of this 18-by-14 rectangle. Show your reasoning.



7.3 Connecting Area Diagrams to Calculations with Decimals

1. Here is an area diagram that represents $(2.4) \cdot (1.3)$.



   a. Find the region that represents $(0.4) \cdot (0.3)$. Label it with its area of 0.12.

   b. Label the other regions with their areas.

   c. Find the value of $(2.4) \cdot (1.3)$. Show your reasoning.

2. Here are two ways of calculating $(2.4) \cdot (1.3)$.

$$\begin{array}{rr} & 2.4 \\ \times & 1.3 \\ \hline & \left.\begin{array}{r} 0.12 \\ 0.6 \\ 0.4 \\ +\quad 2 \end{array}\right\} \text{partial products} \\ \hline & 3.12 \end{array}$$

Calculation A

$$\begin{array}{rr} & 2.4 \\ \times & 1.3 \\ \hline & 0.72 \\ + & 2.4 \\ \hline & 3.12 \end{array}$$

Calculation B

Analyze the calculations and discuss these questions with a partner:

   a. In Calculation A, where do the 0.12 and other partial products come from?

   b. In Calculation B, where do the 0.72 and 2.4 come from?

   c. In each calculation, why are the numbers below the horizontal line aligned vertically the way they are?

3. Find the value of $(3.1) \cdot (1.5)$ in two ways:
    a. Draw and label a diagram. Show your reasoning.

    b. Calculate numerically, without using a diagram. Be prepared to explain your reasoning.

Are you ready for more?

Zhang (JAHNG), or “Chinese yard,” and *li* (LEE), or “Chinese mile,” are two units of length used in China.

1. If 1 li is equal to 150 zhangs, and 1 zhang is approximately 3.645 yards (as used in the United States), about how many yards are in 1 li?

2. There are 1,760 yards in 1 mile (as used in the United States). Estimate how many lis are in 1 mile. Explain your reasoning.

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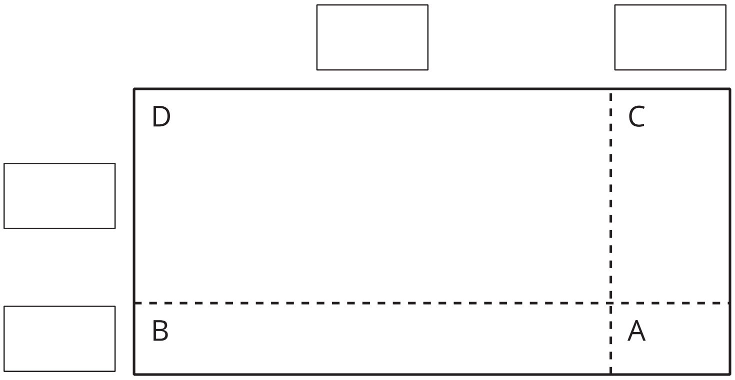
7.4 Using Partial Products

1. Label the area diagram to represent $(2.5) \cdot (1.2)$ and to find that product.



   a. Decompose each factor by place value and write the ones and tenths in the blank boxes.

   b. Label Regions A, B, C, and D with their areas. Show your reasoning.

   c. Find the product that the area diagram represents. Show your reasoning.

2. Here are two ways to calculate $(2.5) \cdot (1.2)$. Each number with a box gives the area of one or more regions in the area diagram.

$$\begin{array}{rrl} & 2.5 & \\ \times & 1.2 & \\ \hline & 0.10 & \boxed{} \\ & 0.4 & \boxed{} \\ & 0.5 & \boxed{} \\ + & 2.0 & \boxed{} \\ \hline & 3.00 & \end{array}$$

Calculation A

$$\begin{array}{rrl} & 2.5 & \\ \times & 1.2 & \\ \hline & 0.5 & \boxed{} \\ + & 2.5 & \boxed{} \\ \hline & 3.0 & \end{array}$$

Calculation B

   a. In the boxes next to each number, write the letter(s) of the corresponding region(s).

   b. In Calculation B, which two numbers are being multiplied to give 0.5?
   Which numbers are being multiplied to give 2.5?

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Lesson 7 Summary

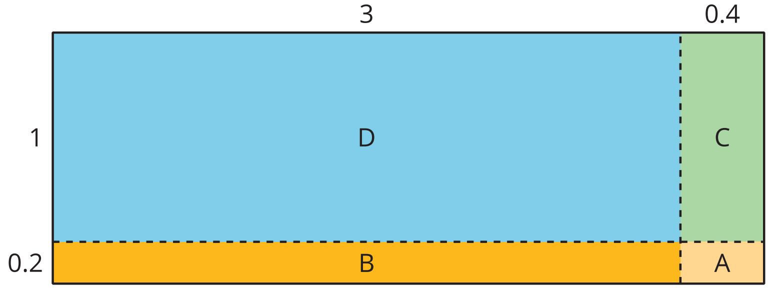
To find the product of two numbers, such as $(3.4) \cdot (1.2)$, we can think of finding the area of a rectangle with those numbers, 3.4 units and 1.2 units, as side lengths.

First, we draw a rectangle and partition each side length by place value, into ones and tenths:

$$3.4 = 3 + 0.4$$

$$1.2 = 1 + 0.2$$

Then, we decompose the rectangle into four smaller sub-rectangles and find their areas.



A: $(0.4) \cdot (0.2) = 0.08$

B: $3 \cdot (0.2) = 0.6$

C: $(0.4) \cdot 1 = 0.4$

D: $3 \cdot 1 = 3$

$0.08 + 0.6 + 0.4 + 3 = 4.08$

Each multiplication gives a *partial product* that represents the area of a sub-rectangle. The sum of the four partial products gives the area of the entire rectangle, 4.08 square units.

We can show the same partial-product calculations vertically. Here are two ways:

$$\begin{array}{lrl} & 3.4 & \\ \times & 1.2 & \\ \hline & \overset{1}{0}.08 & \text{A} \\ & 0.6 & \text{B} \\ & 0.4 & \text{C} \\ + & 3 & \text{D} \\ \hline & 4.08 & \end{array} \qquad \begin{array}{lrl} & 3.4 & \\ \times & 1.2 & \\ \hline & \overset{1}{0}.68 & \text{A} + \text{B} \\ + & 3.4 & \text{C} + \text{D} \\ \hline & 4.08 & \end{array}$$

The calculation on the left shows four partial products, one for the area of each sub-rectangle.

The calculation on the right shows two partial products:

- 0.68 is the value of $(3.4) \cdot (0.2)$, or the combined area of A and B.
- 3.4 is the value of $(3.4) \cdot 1$, or the combined area of C and D.

In both calculations, adding the partial products gives a total of 4.08, which is the product of $(3.4) \cdot (1.2)$ and the area (in square units) of the entire rectangle.

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