



Interpreting and Writing Logarithmic Equations

Let's look at logarithms with different bases.

10.1 Reading Logs

The expression $\log_{10} 1,000 = 3$ can be read as: "The log, base 10, of 1,000 is 3."

It can be interpreted as: "The exponent to which we raise a base 10 to get 1,000 is 3."

Take turns with a partner reading each equation out loud. Then, interpret what they mean.

- $\log_{10} 100,000,000 = 8$
- $\log_{10} 1 = 0$
- $\log_2 16 = 4$
- $\log_5 25 = 2$

10.2

Base-2 Logarithms

x	$\log_2(x)$
1	0
2	1
3	1.5850
4	2
5	2.3219
6	2.5850
7	2.8074
8	3
9	3.1699
10	3.3219

x	$\log_2(x)$
11	3.4594
12	3.5845
13	3.7004
14	3.8074
15	3.9069
16	4
17	4.0875
18	4.1699
19	4.2479
20	4.3219

x	$\log_2(x)$
21	4.3923
22	4.4594
23	4.5236
24	4.5850
25	4.6439
26	4.7004
27	4.7549
28	4.8074
29	4.8580
30	4.9069

x	$\log_2(x)$
31	4.9542
32	5
33	5.0444
34	5.0875
35	5.1293
36	5.1699
37	5.2095
38	5.2479
39	5.2854
40	5.3219

- Use the tables to find the exact or approximate value of each expression. Then explain to a partner what each expression and its approximated value means.
 - $\log_2 2$
 - $\log_2 32$
 - $\log_2 15$
 - $\log_2 40$
- Solve each equation. Write the solution as a logarithmic expression.
 - $2^y = 5$
 - $2^y = 70$
 - $2^y = 999$

10.3

Exponential and Logarithmic Forms

These equations express the same relationship between 2, 16, and 4:

$$\log_2 16 = 4$$

$$2^4 = 16$$

1. Each row shows two equations that express the same relationship. Complete the table.

	exponential form	logarithmic form
	$2^4 = 16$	$\log_2 16 = 4$
a.	$2^1 = 2$	
b.	$10^0 = 1$	
c.		$\log_3 81 = 4$
d.		$\log_5 1 = 0$
e.	$10^{-1} = \frac{1}{10}$	
f.	$9^{\frac{1}{2}} = 3$	
g.		$\log_2 \frac{1}{8} = -3$
h.	$2^y = 15$	
i.		$\log_5 40 = y$
j.	$b^y = x$	

2. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use “?” for the unknown value.
- “To what exponent do we raise the number 4 to get 64?”
 - “What is the log, base 2, of 128?”

Are you ready for more?

How are $\log_{10}(2)$ and $\log_2(10)$ related?

Lesson 10 Summary

Many relationships that can be expressed with an exponent can also be expressed with a logarithm. Let's look at this equation:

$$2^7 = 128$$

The base is 2 and the exponent is 7, so it can be expressed as a logarithm with base 2:

$$\log_2 128 = 7$$

In general, an exponential equation and a logarithmic equation are related as shown here:


$$\begin{array}{ccc} \text{exponent} & & \text{exponent} \\ \downarrow & & \downarrow \\ b^y = x & & \log_b x = y \\ \uparrow & & \uparrow \\ \text{base} & & \text{base} \end{array}$$

Exponents can be negative, so a logarithm can have negative values. For example, $3^{-4} = \frac{1}{81}$, which means that $\log_3 \frac{1}{81} = -4$.

An exponential equation cannot always be solved by observation. For example, $2^x = 19$ does not have an obvious solution. The logarithm gives us a way to represent the solution to this equation: $x = \log_2 19$. The expression $\log_2 19$ is approximately 4.2479, but $\log_2 19$ is an exact solution.