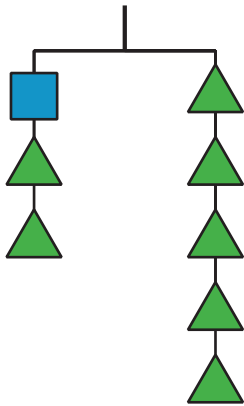


# Unit 4 Family Support Materials

## Linear Equations and Linear Systems

### Section A: Equivalent Equations

This week your student will work on solving linear equations. We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on either side have equal value, just like a balanced hanger has equal weights on either side.



If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance. For example, we could remove 2 triangles from each side of this hanger and it would still balance. We could also add a square to each side and it would still balance.

We can do this with equations as well: Adding or subtracting the same amount from both sides of an equation keeps the sides equal to each other. For example, if  $4x + 20$  and  $-6x + 10$  have equal value, we can write an equation  $4x + 20 = -6x + 10$ . We could add  $-10$  to both sides of the equation or divide both sides of the equation by 2 and keep the sides equal to each other. Using these moves in systematic ways, we can find that  $x = -1$  is a solution to this equation.

#### Here is a task to try with your student:

Elena and Noah work on the equation  $\frac{1}{2}(x + 4) = -10 + 2x$  together. Elena's solution is  $x = 24$  and Noah's solution is  $x = -8$ . Here is their work:

Elena:

$$\begin{aligned}\frac{1}{2}(x + 4) &= -10 + 2x \\ x + 4 &= -20 + 2x \\ x + 24 &= 2x \\ 24 &= x \\ x &= 24\end{aligned}$$

Noah:

$$\begin{aligned}\frac{1}{2}(x + 4) &= -10 + 2x \\ x + 4 &= -20 + 4x \\ -3x + 4 &= -20 \\ -3x &= -24 \\ x &= -8\end{aligned}$$

Do you agree with their solutions? Explain or show your reasoning.



Solution:

No, they both have errors in their solutions.

Elena multiplied both sides of the equation by 2 in her first step, but forgot to multiply the  $2x$  by the 2. We can also check Elena's answer by replacing  $x$  with 24 in the original equation and seeing if the equation is true.

$$\begin{aligned}\frac{1}{2}(x + 4) &= -10 + 2x \\ \frac{1}{2}(24 + 4) &= -10 + 2(24) \\ \frac{1}{2}(28) &= -10 + 48 \\ 14 &= 38\end{aligned}$$

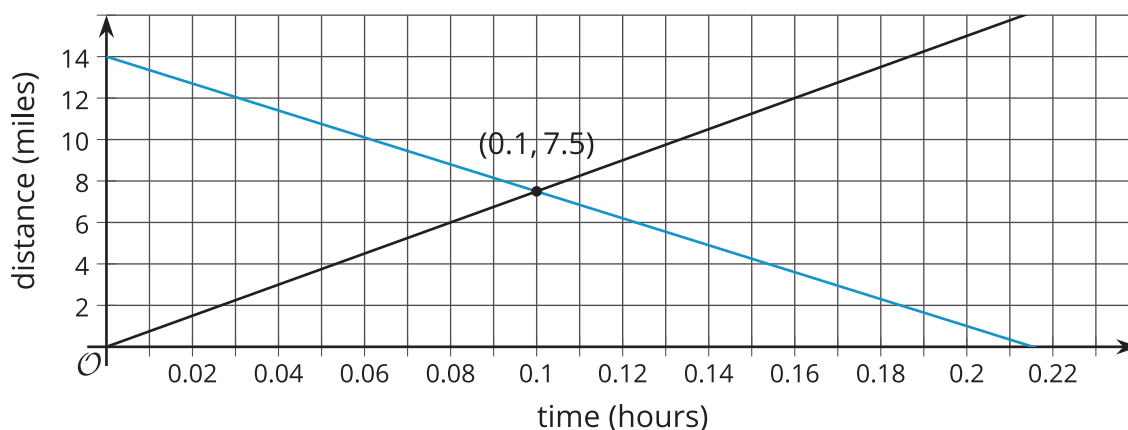
Because 14 is not equal to 38, Elena's answer is not correct.

Noah divided both sides by -3 in his last step, but wrote -8 instead of 8 for  $-24 \div -3$ . We can also check Noah's answer by replacing  $x$  with -8 in the original equation and seeing if the equation is true. Noah's answer is not correct.



## Section C: Systems of Linear Equations

This week your student will work with systems of equations. A system of equations is a set of 2 (or more) equations where the letters represent the same values. For example, say Car A is traveling 75 miles per hour and passes a rest area. The distance in miles it has traveled from the rest area after  $t$  hours is  $d = 75t$ . Car B is traveling toward the rest area, and its distance from the rest area at any time is  $d = 14 - 65t$ . We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is “yes,” then the solution will correspond to one point that is on both lines, such as the point  $(0.1, 7.5)$  shown here. This means that 0.1 hours after Car A passes the rest area, both cars will be 7.5 miles from the rest area.



We could also answer the question without using a graph. Because we are asking when the  $d$  values for each car will be the same, we are asking for what  $t$  value, if any, makes  $75t = 14 - 65t$  true. Solving this equation for  $t$ , we find that  $t = 0.1$  is a solution, and at that time the cars are 7.5 miles away because  $75t = 75 \cdot 0.1 = 7.5$ . This finding matches the graph.

### Here is a task to try with your student:

Lin and Diego are biking the same direction on the same path, but start at different times. Diego is riding at a constant speed of 18 miles per hour, so his distance traveled in miles can be represented by  $d$  and the time he has traveled in hours by  $t$ , where  $d = 18t$ . Lin started riding a quarter hour before Diego at a constant speed of 12 miles per hour, so her total distance traveled in miles can be represented by  $d$ , where  $d = 12\left(t + \frac{1}{4}\right)$ . When will Lin and Diego meet?

Solution:

To find when Lin and Diego meet, that is, when they have traveled the same total distance, we can set the two equations equal to one another:  $18t = 12\left(t + \frac{1}{4}\right)$ . Solving this equation for  $t$ ,  $18t = 12t + 3$ ,  $6t = 3$ ,  $t = \frac{1}{2}$ .

They meet after Diego rides for one half hour and Lin rides for three quarters of an hour. The

distance that they each travel before meeting is 9 miles, because we can use  $\frac{1}{2}$  for  $t$  in Diego's equation and see that  $9 = 18 \cdot \frac{1}{2}$ . Another way to find a solution would be to graph both  $d = 18t$  and  $d = 12\left(t + \frac{1}{4}\right)$  on the same coordinate plane and interpret the point where these lines intersect.

